

Confuses numbers when counting in twos; has difficulty understanding that a pair consists of two objects

Opportunity for: solving problems in a real-life context



1 YR x/÷

Wave 3 mathematics

multiplication and division

Tracking back to Year R

Resources

- Obvious pairs of socks and gloves, some odd socks
- Box
- Large 100-square
- Number cards (Resource sheets 1 and 2)

Key vocabulary

- pair
- count in twos
- go together
- match

Teaching activity

Time 10–15 minutes

‘Look at all these things. We’re going to make some pairs and you will be learning to count in twos.’

? What do you think it means to have a pair of something?

Clarify that things make pairs because they match and have the same colour or pattern. Let the child find the pairs and identify the odd socks.

If the child is unsure about matching patterns or cannot identify what makes a pair, identify what is the same in a pair, for example, a blue zigzag pattern on mittens. Use some odd socks to illustrate and clarify what isn’t a pair.

? How many things make up a pair?

Clarify that there are two objects in a pair. For example, we have two feet, so we need two matching socks.

? Can you count the pairs?

If the child is uncertain about counting the pairs, move on to counting in twos, because this is a more important concept at this stage.

? Can you count the socks?

If the child counts the socks in ones, support counting in twos.

Help the child to recite ‘two, four, six, eight, who do we appreciate?’ (A team-game chant.)

Ask the child to put out number cards 2, 4, 6, 8, and so on.

Extend the counting in twos to about ten or twenty and support the child in learning to recite these numbers.

Link the chant to numbers on a 100-square.

? What did you learn today?

Spotlight 1

Confuses numbers when counting in twos; has difficulty understanding that a pair consists of two objects

Opportunity for: reasoning about numbers

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Two, four, six, eight

Time 5–10 minutes

Resources

- Cubes or small objects, for example dinosaurs, toy cars
- Box
- Number cards (Resource sheets 1 and 2)

Key vocabulary

pair	how many altogether?
count in twos	two more
order	

Teaching activity

‘Today we will be using some dinosaurs (or other objects) to count in twos.’

Support the child in putting about ten of the objects in twos.

? Can you count them in twos?

If the child falters on this, or counts in ones, reduce the objects to just six or eight and show how to move and count the objects, saying ‘two, four, six’. (See also 1 YR +/-.)

Ask the child to copy what you did. Establish that moving objects as we count them can help us to get the right number.

? How many objects are there altogether?

? If we take two more objects out of the box, how many are there now?

With the number cards, ask the child to point out the numbers we say when we count in twos.

? Can you say each number and point to the right number card as you say it?

Muddle up the number cards.

? Can you put the number cards in the right order?

Finish by asking the child to recite the numbers in twos as far as they can without looking at the number cards or objects.



‘Let’s all count in twos and see if we can get to three hundred and fifty.’

Spotlight 2

Confuses numbers when counting in twos; has difficulty understanding that a pair consists of two objects

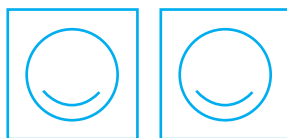
Opportunity for: solving practical problems

Smile, please!

Time 5–10 minutes

Resources

- Several smiley faces without eyes



Key vocabulary

pair

count in twos

Teaching activity

‘We are going to use these smiley faces to find out more about pairs and counting in twos.’

? How many eyes have you got?

Explain about pairs of eyes. Ask the child to draw pairs of eyes on an appropriate number of smiley faces.

? How many eyes are you going to put on each face?

? Can you count the eyes in twos?

Support pointing at the faces and counting in twos. If appropriate, show some more smiley faces, ask the child to draw pairs of eyes, then count all the eyes in twos again.

? How many pairs of eyes are there?

Establish that a pair always has two things, so to count the pairs we can count the smiley faces.

? How many eyes are there?

If the child is still struggling with pairs, support them in finding more pairs over the next few weeks during ordinary classroom activities.

It is important to move on to counting in twos and not spend too much time on pairs.

Finish by asking the child to count in twos as far as they can.

Spotlight 3

Confuses numbers when counting in twos; has difficulty understanding that a pair consists of two objects

Opportunity for: looking for patterns

Whisper and shout

Time 5–10 minutes

Resources

- A5 pieces of paper with numbers 1 to 10 or 20 or 30 on them
- Red circle (or a zero)
- Pieces of string or rope
- Sticks, such as lolly sticks

Key vocabulary

words for counting in twos

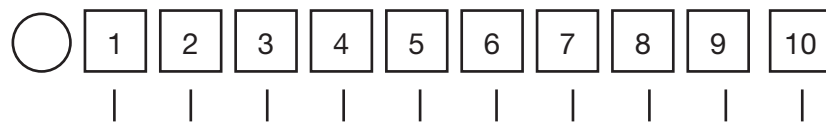
Teaching activity

(It is fun to go out of the class to do this activity, so that the numbers can be shouted!)

Explain to the child that this activity will help them to learn to count in twos along a number line.

? How far can you count in twos?

Lay out paper numbers on the floor with a red circle at the zero space.



Ask the child to stand on the red circle and step onto the paper with 1 on it, saying 'one' in a whisper. Then step onto the 2 and say that loudly. Continue counting in twos loudly, whispering the numbers in between.

If the child is finding this hard, support them by counting with them. The child might need to repeat the activity a few times over the next few days.

Ask the child to go back to the red circle and jump along the number line, landing only on the numbers 2, 4, 6, 8, and so on, and say those numbers out loud.



Then ask them to do it again, explaining that you will count the number of jumps they do in your head.

'You did five jumps of two and landed on 10.'

Help the child to record their jumps on a drawn number line.

? Without looking at the number line, count in twos again and see how far you can go.



Back in class you could ask everyone about the number of jumps of two it takes to get to 30, 50, and so on. Encourage the children to make their own number line with number cards to 50 or 100 (this needs a few tables to be put together, or do it on the floor) and to count the jumps of two.

Spotlight 4

Confuses numbers when counting in twos; has difficulty understanding that a pair consists of two objects

Opportunity for: real-life context

Two pence counting

Time 5–15 minutes

Resources

- Bag of 2p coins
- Pictures, such as lollipops, labelled 2p



Key vocabulary

words for counting in twos
how much?

Teaching activity

Explain to the child that this activity will help them to count in twos with pennies – something they will find useful when they use money to do shopping.

? What do you know about counting in twos?

Let the child demonstrate what they know and, from that, decide how many 2p coins to use.

Show the pictures of the lollipops.

? How much are the lollies? Can I buy one with this coin?

Let the child pretend to buy lollies with 2p coins. Then count the coins in twos.

If the child struggles with this, keep to just a few coins and try to move them on to be able to deal with five or six coins.

If the child is secure with this, encourage them to move on to deal with larger numbers.

? How many lollies would we need if we wanted to buy one for everyone in the class?

Count out the appropriate number of coins and support counting them in twos. Encourage the child to move the coins as they are counted and you can put them in piles of ten pence; that is, five 2p coins together in each group.



‘We have been counting lots of 2p coins.

We counted in twos, then we put them in groups of ten pence.

Why do you think we did that?’

Spotlight 5: a learning check

Confuses numbers when counting in twos; has difficulty understanding that a pair consists of two objects

Opportunity for: explaining and discussing

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Pairs, twos and bees

Time 10–15 minutes

Resources

- Box with a variety of counting objects not used in the previous activities, for example shoes, bottle tops, shells, paint brushes
- Large 100-square
- Number cards (Resource sheets 1 and 2)

Check: does the child use key vocabulary?

pair	count in twos
go together	match

Teaching activity

Explain that the activity is about finding pairs and counting in twos.

? Can you put some of these things in pairs?

You might have to support the child at first in finding pairs, for example matching up a pair of two silver bottle tops, and so on.

? How many red beads do you need to make a pair?

? Why did you put those two objects together? Do they make a pair?

Accept explanations of pairs as long as there are two items put together – even when explanations seem a bit bizarre!

? Can you count how many pairs you have made?

Observe if the child counts each pair as one pair.

? How many objects are there altogether here? Can you count them all?

If the child counts these in ones rather than in twos, prompt counting in twos, demonstrating if the child still seems confused. It often helps to move each pair as it is counted.

? Can you find those numbers (two, four, six, eight, and so on) on cards and show me where they are on the 100-square?

Observe if the child can miss out every other number and note how far they can count up to in twos.

? Can you explain to me what a pair is?

? Explain to me why you think it is useful to count in twos.



If the child still needs more help in counting in twos, suggest making a swarm of bees over the next few days and weeks, drawing simple shapes on scrap paper and giving each bee two wings. Others can join in and the bees can be made at home, then brought into school. Try to get beyond fifty bees and put up the display where you can add to it over the next term or two.



Learning outcomes

By the end of this set of activities, children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- count in twos;
- identify a pair as having two matching objects;
- count a pair as one unit (this is not an essential aspect of multiplication and division so you can just ask the child to find pairs in everyday situations over the next few weeks);
- solve some practical problems.



Line up in twos

Time 10–20 minutes

Confuses numbers when counting in twos; has difficulty understanding that a pair consists of two objects.

Resources

- Blank loop track (Resource sheet 21)
- Cubes or pairs of animals
- 1–6 dice

Check: does the child use key vocabulary?

- pair
- count in twos
- how many twos?

Teaching activity

‘Today we are going to play a game, **Line up in twos**, to help you with counting in twos.’

Prepare *Blank loop track* (Resource sheet 21) perhaps with some stickers or some decoration related to the child’s interests or a class theme connected with pairs if possible.

Children can cooperate in pairs or race each other.

Each player or pair needs their own game track.

How to play

1. Players take turns to throw the dice and put out that many twos – cubes or animals. So if 3 is thrown, three pairs of cubes are put out, six altogether.
2. Each pair of cubes / animals fits on a space on the track and at the end of each turn, each player counts their cubes in twos, ‘two, four, six.’
3. On a player’s second turn, when they count their twos, they count from the beginning each time, ‘two, four, six, eight, ten’ and so on.
4. When the player has a pair of cubes / animals on each space on the track, encourage them to count in twos without help.

Variations:

1. Instead of playing on the track, play by making a long line across a table, and compete to see who can get the longest line of pairs of cubes after five goes each.
2. At the end of the game combine all sets of twos and see if they can count how many altogether in twos. This number can be identified on a number line and 100-square.

Learning outcomes

By the end of a set of related activities children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- count objects in twos.

Has difficulty with identifying doubles and adding a small number to itself, for example $2 + 2$, to make twice as many

Opportunity for: reasoning about numbers

Resources

- Dominoes
- Round counters
- Ordinary six-sided dice
- Number cards (Resource sheets 1 and 2)
- + and = cards (Resource sheet 8)

Key vocabulary

- double
- adding a number to itself

Teaching activity

Time 10–15 minutes

Explain to the child that you are going to do an activity with dominoes and it will help them to find out more about doubles.

? Tell me what you think 'a double' means.

Talk about familiar uses of the word 'double', for example a double-decker bus, a double bed, a double-breasted jacket.

Invite the child to lay out the dominoes, face up.

? Can you find any doubles dominoes?

If the child finds this difficult, pick out a couple of doubles and support the counting of the dots, clarifying that each part of the domino has the same number of dots, making a double.

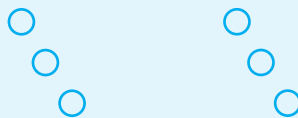
? I've got double three and double two here. Can you find another double?

If the child is struggling, put away most of the dominoes, leaving the doubles and a few others. Find a non-double and explain how the number of dots is different on each part of the domino. Compare this with a double.

? Why is this domino a double? Why is this not a double?

When you have all the doubles, count the dots to clarify that the double two has two dots then another two dots.

If the child is struggling with this, use round counters to lay out the dot patterns, carefully copying the domino pattern and counting how many counters are in each pattern.



'Three added to itself, three add three. Double three is six dots altogether.'

You could put the dominoes in order from double one to double six. You might choose to leave out double zero.

With number cards and + and = cards, and/or with paper and a pen, make some doubling number sentences.

? This domino is two and another two. How shall I write that to make a number sentence?

Encourage the child to tell you what to write.

Double 2 is 4

$2 + 2 = 4$

? Now can you choose a double domino and use these cards to make a number sentence?

Support the child in making the sentences with cards (or with you recording them) for as many of the numbers as you can while the child is able to concentrate. Encourage the child to use the language explaining 'a double is a number added to itself'.

Demonstrate using counters again.

? If I have four counters and then three counters next to them, does that make a double? How could you make it a double? Which of our number sentences goes with that double?

If the child is struggling with the number sentences, model them in a different way, for example with fingers, cubes or bottle tops. 'I've put out four cubes. Now you make that into double four.'

Let the child throw a dice and tell you the number of dots, and then work out the double of that any way they choose.

This might be a good time to ask the child to record what they have done in any way they choose. This can give you an insight into how they are thinking.

? Can you tell me what you were thinking when you drew it that way?

If the child is struggling, you could turn the task into a game where you throw the dice but say the double number wrongly. For example, if you throw a three, put out another four counters and say 'double four is eight', encouraging the child to point out that the dice only showed three dots, so you didn't make a double.

Help the child to say the doubles up to double six, making the number sentences to match.

? What did you learn today?



You could display the doubles sentences for a few days and add some more challenging ones for the whole class, particularly double twenty-five, thirty, fifty and a hundred. Ask who can say what a double is, aiming to get a response such as: 'to double a number you add a number to itself'.

Borrow lots of boxes of dominoes for wet playtimes and plan a class tournament.

Spotlight 1

Has difficulty with identifying doubles and with adding a small number to itself, for example $2 + 2$, to make twice as many

Opportunity for: developing mental images

Double Dudley

Time 10–15 minutes

Resources

- Dice and/or number cards to suit the child (Resource sheets 1 and 2)
- Bag or box
- Cubes or sweets

Key vocabulary

- double
- adding a number to itself

Teaching activity

Explain that today you are going to play a game called **Double Dudley** which will help them to learn more about doubles. Throughout the activity it would be helpful to ask the child about the pictures they have in their mind.

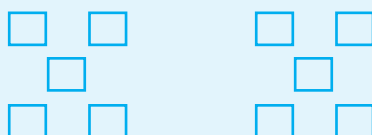
Each player needs to write down the numbers that are possible with the cards/dice they are using, so for cards 4–10 there are seven possible numbers to double.

Possible doubles						
4	5	6	7	8	9	10

How to play

1. The aim of the game is to throw the dice, or take a number card out of a bag (so it isn't possible to see numbers until they are taken out), and say double that number as quickly as possible. (Putting the card back in the bag makes the game last longer and keeps the same degree of chance.)

Support any numbers which the child is unsure of with cubes or sweets, laying out the doubles clearly.



? If you shut your eyes, can you see a picture of five added to another five?

2. At the end of their go, the player ticks the number they doubled. The winner is the first player to tick all their numbers. (So there is a large element of luck.)

Encourage the child to 'see' the picture of the cubes in their head.

? What did you do to work that out?

Variation

- Other doubles to practise include fifteen, eighteen, twenty, twenty-five and fifty.

 When the child is confident with doubling all the numbers fairly quickly, you could invite another child to play. This can be a class game in small groups. You can keep to the same rules at first, but adapt the game as confidence grows so that the winner is the child who says the double first. That child wins a point if they are correct.

Spotlight 2

Has difficulty with identifying doubles and with adding a small number to itself, for example $2 + 2$, to make twice as many

Opportunity for: reasoning about quantities

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Twice as much as Twiggy

Time 10–15 minutes

Resources

- Two pieces of ribbon or string, one half the length of the other
- Scrap paper
- Scissors
- Sticky tape
- Two beakers
- Jug of coloured water

Key vocabulary

about twice as much
about twice as long/high

Teaching activity

If you want to, you can set this activity in the context of twins, Twiggy and Tiny. Twiggy always wants twice as much of everything as Tiny.

‘At the end of the activity I am going to ask you what you learned about “twice as much”.’

? Can you tell me which bit of ribbon/string is twice as long as the other bit? So which bit would Twiggy want?

If the child struggles with that, lay both bits alongside each other and demonstrate how you can fit two of the shorter bits against the longer one.



Support the child by talking about how ‘you need two of the shorter bits to make the same length as the longer bit’, and ‘the longer bit is twice as long as the shorter bit’.

With the two empty beakers, invite the child to put a small amount of water in one beaker for Tiny and about twice as much in the other for Twiggy.

If the child needs more help with ‘twice as much’ quantities, try cutting a short paper strip and ask the child to estimate about how long another strip would have to be to be twice as long.

Put a longer paper strip on a door with sticky tape and ask the child to put their hand about where a strip would come if it was twice as high.

? What did you learn today about ‘twice as much’?

Spotlight 3

Has difficulty with identifying doubles and with adding a small number to itself, for example $2 + 2$, to make twice as many

Opportunity for: developing mental images

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Hungry Horatio

Time 10–15 minutes

Resources

- Cubes, bricks
- Modelling material to make model sausages

Key vocabulary

double
adding a number to itself
about twice as much

Teaching activity

You can set this activity in the context of Hungry Horatio, who always eats twice as much as his brothers and sisters. The activity will be about finding twice as much of several things.

‘Let’s pretend these are sausages. Everyone has three sausages, but Hungry Horatio wants twice as many.’

? What number is twice as much as three?

Support the child by reminding them of doubling. Say that we need three and another three. If we add three to itself we end up with twice as many; that is, six.

If the child needs more help, repeat the activity with cubes – brown ones for buns, white ones for sandwiches, red ones for apples, and so on, using numbers to suit the child.

? If Horatio’s brothers and sisters each have ten cherries, how many will Horatio want?

? Can you show me how you are working that out?

Encourage the child to ‘see’ pictures in their mind and to use the key vocabulary (above).

‘Horatio’s little brother is six today and has six candles. When it is Horatio’s next birthday he will be twice that age.’

? How many candles will Horatio have?

? Can you make a picture of them in your head?

If the child is finding it hard to make pictures in their head, suggest that they make a drawing of what they are doing. Then ask them to close their eyes and try to see the drawing in their head. When children make their own recordings it can give us insight into what they are thinking.

? What can you tell me about the word ‘double’ and the phrase ‘twice as much’?

Spotlight 4

Has difficulty with identifying doubles and with adding a small number to itself, for example $2 + 2$, to make twice as many

Opportunity for: investigating a pattern

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Calculator quits

Time 10–15 minutes

Resources

- Circles of coloured paper or plates
- Cubes, some of them in twos and others in tens 'trains' (or other support for counting on)
- Large clear calculator
- Wipe-clean number line
- 100-square

Key vocabulary

- double
- adding a number to itself

Teaching activity

'This activity is going to be about doubling numbers and we are going to see if we can make some really big numbers.'

Lay out some of the circles and put two cubes on the first one.

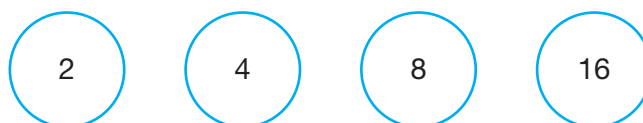
? What number is double two?

? How did you work that out?

? Can you write four on that circle?

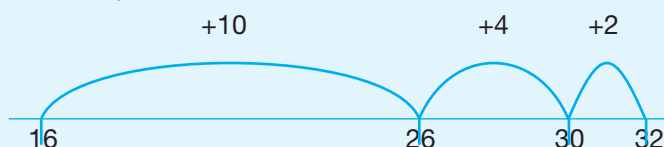
If the child struggles with the doubling, remind them of a domino with two dots on each part, making four dots altogether. 'So double two is four. Four is twice as much as two.'

Continue by asking the child to double four, and so on, counting out the cubes in twos and putting the cubes on the plates.



Once the child starts to find the numbers more challenging, use tens 'trains' of cubes (or any other equipment which the child is used to) to aid at least one calculation beyond their usual limit, and also show the 'jump' on a number line.

Try not to rely on the child counting cubes in ones because this can lead to long-term difficulties with larger numbers. Focus on the jump of ten from sixteen to twenty-six, reminding the child about adding ten on a 100-square.



‘Sixteen and another sixteen is thirty-two.’

Once the numbers are as large as you think the child can cope with, the child can ‘quit’ after writing down the last number they could make, for example thirty-two, and they can use a calculator to find the next few doubles. Observe the child very carefully as they use the calculator.

? Can you show me how to find double thirty-two on a calculator?

Focus on needing to add a number to itself to find a double.

$(32 + 32 = 64)$

? Do you know another way to find twice as much as thirty-two on a calculator?

Don’t worry if they don’t! It might be interesting to see if any come up with thirty-two multiplied by two.



Challenge the class to go on with this doubling chain and display the numbers. Some can start with three and do a doubling chain. This can make an interesting homework challenge for parents to join in.

Spotlight 5: a learning check

Has difficulty with identifying doubles and with adding a small number to itself, for example $2 + 2$, to make twice as many

Opportunity for: explaining and discussing

I can double

Time 10 minutes

Resources

- A range of things to double that haven’t been used in other activities, for example coloured string or knitting wool, different coins, cups and plastic teapot, small toys
- Scissors

Check: does the child use key vocabulary?

double
adding a number to itself
about twice as much/long/high

Teaching activity

‘Look at all these things. I want you to show me how you can double things. Choose something to start with.’

If the child isn’t responding well, choose something like 1p coins, or some things to count.

Let the child explain what they are doubling.

? I’ve got three frogs here. How many frogs would be double three?

? If your brother had four pence and you had double that, how many pence would you have?

Establish that would be eight pence and put out the coins.

? Who had twice as much, you or your brother?

? Can you put out the coins for double nine pence? Double twelve pence?

? What do you think is the largest number you can double?

If the child sticks with quite small numbers, suggest doubling fifty, or a hundred, or a thousand, or a million.

? Can you explain to me what a double is?

? Do you have a favourite way of finding a double in your head?

? How could you write a number sentence for doubling ten?

Move on to finding double quantities.

? Can you cut a bit of wool as long as your hand and another bit about twice as long?

? Can you put some water for me in this teacup and put about twice as much for you in that one?

? Can you explain to me what you did to find twice as much?

? Which bits of our work on doubling do you think you did really well?

? Is there anything about doubling that you would like more help with?



It can be difficult to find doubles of quantities in balance scales, but you can challenge children to find two books that weigh about as much as one larger book, or two objects that balance one larger one. (Take care, though, not to say that large things weigh more. Remember to show children that a small pebble can balance a large piece of polystyrene.)

Learning outcomes

By the end of this set of activities, children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- identify a double as a number added to itself;
- know some doubles of numbers by heart;
- write a simple doubling number sentence;
- understand how to find double or twice as much of quantities;
- solve practical problems with doubling.

Makes unequal groups and is unable to compare the groups

Opportunity for: reasoning about numbers

Resources

- Different coloured cubes
- Number cards (Resource sheets 1 and 2)
- Three sizes of paper with smiley faces or stick people drawn on them in three different sizes (it can help to have sad faces to show unfair sharing)

Key vocabulary

equal groups	fewer
more	same number
less	too many

Teaching activity

Time 10–15 minutes

Explain to the child that today they are going to pretend to go on a picnic and that the different coloured cubes are different kinds of food, for example, blue cubes are blueberry muffins. The activity today will help them to make equal groups.

? What do you think it means to make equal groups with these cubes?

Talk through with the child what they do with the cubes. Encourage them to make groups larger than two cubes.

If the child struggles with sharing, teach ‘one for you, one for me’.

? How many have you got? How many have I got? Is it fair?

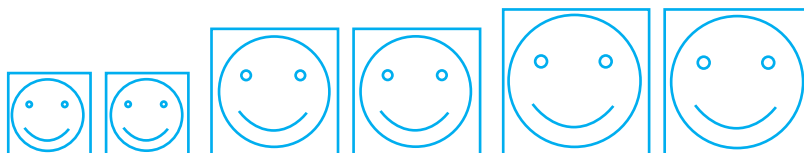
Explain that they are going to pretend that the white and brown cubes are sandwiches, and they are going to share them out to pairs of people at a picnic. But the bigger people eat more and are very hungry so they should have more sandwiches than the smaller people.

Talk about how many sandwiches the people might eat, for example, three each for the smaller people, five each for the middle-sized people and six each for the bigger people.

Explain that each person in each pair must have the same number of sandwiches to make it fair, and also that there might be lots of sandwiches left over in case some extra people come to the picnic.

Deal with just one pair of people at a time.

? How many sandwiches are you going to give to the pair of smaller people? Have they both got the same number? How do you know?



If the child is struggling with this, help them to ‘move and count’ and to say how many sandwiches that person has. You might find it helpful to write the number of sandwiches or place the relevant number card next to the person.

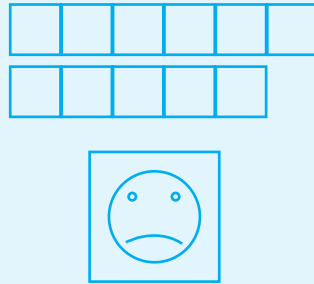
? This person has three sandwiches, so how many shall we give to this other person who is the same size?

Explain that it must be fair. These two people of the same size need the same number of sandwiches.

If the groupings are still unequal, encourage the child to line up the cubes and to compare them.

'This person has more and this person wants the same number to make it fair.

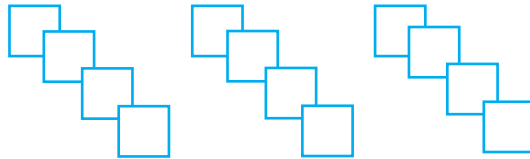
Look, she is sad.'



? How many more do you need to give her?

If the child is still unable to compare the groups and talk about the numbers being the same or fair, you might want to try the activity again at a later date with some real sweets to share fairly with another child.

Put out some equal groups of cubes using a colour not used above, maybe three groups with four cubes in each group.



? What can you tell me about these three groups?

Spotlight 1

Makes unequal groups and is unable to compare the groups

Opportunity for: exploring a real-life context

Pennies in purses

Time 10–15 minutes

Resources

- Twelve 1p coins and four containers
- Four smiley faces on paper (all the same size)
- Small world people, character toys

Key vocabulary

equal groups	fewer
more	same number
less	too many

Teaching activity

‘We are going to do a sharing activity today with 1p coins and containers, so that the people all have an equal number of 1p coins and it is fair.’

Put out three people (smiley faces) with a container each.

? How many people are there? How many containers?

? How many containers does each person have?

Take six 1p coins and ask the child to put them in the three containers so that each person has the same number.

If this proves too challenging, help them to count the 1p coins.

‘I think this person has fewer than that person. Let’s count them carefully.’ (Use ‘move and count’.)

? Does this person have more than that person?

? How can you make each person have the same number of 1p coins?

If you have time, move on to either having four people and eight or twelve 1p coins, or keep three people and use nine 1p coins. You could ask the child to record their work in any way they want. This can give you an insight into their thinking.

? Do you think you were successful in making equal groups today?

Spotlight 2

Makes unequal groups and is unable to compare the groups

Opportunity for: developing mathematical language

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Comparing cod

Time 10–15 minutes

Resources

- Three toy cats or penguins or bowls
- Nine, twelve or fifteen toy fishes

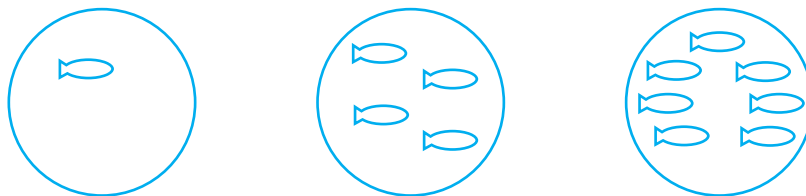
Key vocabulary

equal groups	fewer
more	same number
less	too many

Teaching activity

Choose some kind of context for the activity, for example, penguins being fed at the zoo or cats and their food bowls. Display three toy penguins or cats or bowls with nine or twelve or fifteen fish in total, making each group unequal.

‘We’re going to be working on making equal groups again today. I’m sharing out these twelve fishes, so that each cat/penguin will get an equal amount. Then that will be fair.’



Listen carefully to what the child says and develop your questioning in line with their response. You are trying to use a wide range of comparing words, for example ‘fewer’, ‘more than’, ‘too many’, and so on (see key vocabulary above).

? Do you think the penguins/cats will think that is fair? Why not?

? Can you tell me which penguin/cat has the fewest fishes?

? Can you point to a group that has less than this one?

? Can you point to a penguin/cat that has more than this one?

? How could we make equal groups now?

If the child is finding the sharing and comparing too challenging, repeat the activity in a different context, for example, feeding dolphins, or putting biscuits on plates.

Support the child in making equal groups, establishing that each group must contain the same number.

? Can you tell me what you are doing as you move the fishes?

Encourage the child to count carefully, doing ‘move and count’ if they get muddled.

? Can you tell me about these groups now? Are they equal?

Ask the child to make a picture or some other recording of what they have done today.

? What did you learn today?

? What would you like more help with?

Spotlight 3

Makes unequal groups and is unable to compare the groups

Opportunity for: reasoning about numbers

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Apples on the plate

Time 10–15 minutes

Resources

- Four paper plates
- Twenty apples or cubes
- Four small world people or character toys or smiley faces

Key vocabulary

equal groups	fewer
more	same number
less	too many

Teaching activity

‘Today we are going to do an activity with these apples and plates to learn more about putting things into equal groups.’

You could set it in a real context, for example, sharing fairly between children or feeding apples to horses or to monkeys at the zoo.

Put out three plates and count the twelve apples with the child.

? Can you make a guess at how many I can put on each plate to share the apples out fairly?

You can start to distribute the apples unequally on the plate, putting far too many on the first plate.

? Do you think we have enough apples to give everyone this many?

Ask the child to share out the twelve apples fairly.

If the child needs help with making equal groups or comparing the groups, work with them to count and compare the numbers using the key vocabulary. (See above.)

? This plate has eight apples and that is too many. This plate only has three. Could you help me to take some from this plate and put them with this apple to make the groups equal so it is fair?

? Which plate has fewer than this plate?

? Can you make it fair?

When the three plates have four apples each, encourage the child to count and to talk about and explain what they have done.

? Can you tell me if the apples are all now in equal groups?

? How many are in each group?

? How many groups are there? How do you know?

Repeat the activity, working with four plates. Use twelve apples or cubes again for children who need that support, or move on to using sixteen or twenty apples or cubes.

? Does each plate have the same number? How can you be sure?

If there is time, ask the child to choose their own number of plates and to choose a number of apples that they think will go into equal groups on those plates.

? **What did you learn today about equal groups?**

Spotlight 4

Makes unequal groups and is unable to compare the groups

Opportunity for: solving practical problems

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Knock knock

Time 10–15 minutes

Resources

- Paper plates
- Twelve pretend ‘chocolate chip cookies’

Key vocabulary

equal groups	fewer
more	same number
less	too many

Teaching activity

‘We’re going to share out these twelve chocolate chip cookies so that you and I have an equal number.’

? **Can you put them in two equal groups?**

If the child struggles with that, keep encouraging the child to count, using words such as ‘too many in that group’ and ‘all the groups need to have the same number’.

? **Now we have six for you and six for me. Are they in equal groups? How do you know?**

Then knock on the table.

‘Jed is at the door and he has come to see us. Let’s give him some of the cookies.’

? **Can you share them into equal groups for three people?**

? **How many have we each got now?**

? **Are you sure they are in equal groups?**

Knock on the table again. ‘Oh look, Ali has come as well.’

? **What can we do so that Ali can have an equal share with us?**

? **How many people now want to share the cookies?**

? **How can you be sure that the cookies are in equal groups?**

? **If we put the cookies into equal groups of two, how many people could have two cookies each?**

Ask the child to tell you something they have learned today about putting twelve into equal groups.



How many different ways can you find to put twelve objects into equal groups?

Spotlight 5: a learning check

Makes unequal groups and is unable to compare the groups

Opportunity for: explaining and discussing

Cherry buns

Time 10–15 minutes

Resources

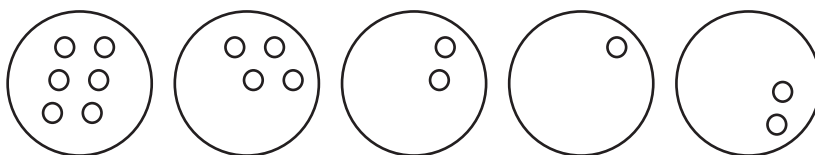
- Round counters or beads (red and other colours)
- Play buns or circles of paper
- Two sizes of sweets or small objects such as linking cubes
- Number cards (Resource sheets 1 and 2)

Check: does the child use key vocabulary?

equal groups	fewer
more	same number
less	too many

Teaching activity

Prepare five buns and distribute fifteen cherries (red counters) unequally on them while explaining that today we are going to pretend we are bakers who make cherry buns and we must put an equal number of cherries on each bun. But look at the buns! Something is wrong.



? What's wrong with this bun? (You are looking for the child to say there are too many cherries on the bun – so the cherries aren't in equal groups.)

? Do you think you could put the cherries equally on the buns?

Ask the child to tell you what they are doing as they do it. Listen to see if the child can talk through the activity using the key vocabulary.

? How many cherries are on each bun now?

? Are the cherries in equal groups?

If the child still seems unsure about equal grouping, ask them to count each red counter carefully and ask them, using circles of paper, to draw an equal number of cubes (cherries) on each bit of paper so that it is fair and so that they have made a picture of the buns. Ask the child either to find the right number cards for each circle, or to write down the correct number of cherries.

With two circles of paper, display about four large blue counters/sweets on one bit of paper and about seven small blue counters/sweets on the other.

Talk through whether this would be a fair way to share sweets – giving four large ones for one person and seven small ones for the other person. (This could be a fair way to share food, with someone having more smaller items. And some people need more food than others.) Then focus on the *number* of cubes.

? Does one person have more sweets than the other when you count them?

Encourage the child to count and talk about the numbers and establish that one person has more when you count the number of sweets.

Remove the small cubes/sweets and ask the child to put out enough large cubes/sweets for each person to have the same number of the same size.

? **What are you trying to do?** *(You are looking for the child to talk about having the same number in each group.)*

? **Can you explain to me what putting things into equal groups means?**

If this proves too challenging for the child to explain, ask them to make unequal groups with counters and to tell you how they know the groups are unequal. Then ask them to move the counters to equal groups and to explain what they are doing.

Learning outcomes

By the end of this set of activities, children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- put objects into equal groups and explain what they are doing;
- use the language of comparison to compare groups and say which ones have too many or too few items;
- apply mathematics in simple real-life contexts.

When sharing, can sometimes make equal groups but has no strategies to deal with any left over

Opportunity for: developing mathematical strategies

Resources

- Items for sharing, for example, real biscuits, hard sweets, cubes, chocolate bars, pens

Key vocabulary

fair	predict/guess
share out	share between two/three
left over	groups
same number	

Teaching activity

Time 10–15 minutes

‘Today we are going to do some fair sharing with cubes/sweets and biscuits. This activity will help you to share fairly and find out what to do when things are left over.’

? What do you think fair sharing means?

? Can you share these seven cubes/sweets equally between you and me?

Help the child to talk through the sharing.

If the child has no strategies at all for sharing, try out ‘one for me, one for you’ sharing.

? How many do we have each? Is it fair?

Help the child to count how many there are in each group.

If the child has not shared fairly and the groups are unequal, talk about being fair, then do some more sharing with real items until the child has grasped that everyone must have the same number of items.

When the objects are in equal groups, talk about how fair it is now that everyone has the same number. You could draw two smiley faces.

? What about this one sweet/cube left over?

Listen carefully to the child and build on their ideas. If the child seems to want to do something with the left-over sweet/cube, try sharing between two using seven biscuits or something else that can be broken easily.

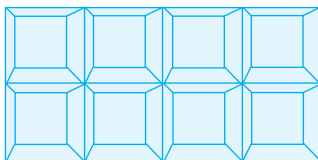
? What could you do this time with the one biscuit left over?

Make the point that you can break a biscuit to share it fairly, but you can’t cut a hard sweet in two very easily and you mustn’t break the school cubes!

Break the biscuit into two if the child doesn’t suggest that.

Help the child to come to some real-life solutions. You could break the biscuit into two bits, or one of you could have an extra one, or you could put it back in the tin.

Repeat the activity with other items that can be split, such as a bar of chocolate.



? Can you share this bar of chocolate fairly between five people?

? How much would they get each?

? What could we do with the chunks of chocolate left over?

Talk with the child about splitting the chocolate between different numbers of people.

If the child is having difficulty with this, you might find it helpful to ask them to draw pictures, and/or write, or you write for them, to record what they have been doing. This can give you insight into their thinking. If you feel they need extra help, you could repeat the activity another day with a bar of chocolate that is divided differently from the one above.

If you have time, do another activity with items you can't split, for example, pens.

? What did you learn today about sharing fairly and left-over items?

The child can record what they did in their own way, perhaps with just one of the items used.



You could make a list with the whole class of different strategies to deal with left-over items. You can cut food, put biscuits back in the tin, put money back in a purse, agree to share things like pens. What else?

This can make a useful homework task – 'How we share things fairly in our family'.

Spotlight 1

When sharing, can sometimes make equal groups but has no strategies to deal with any left over

Opportunity for: exploring a real-life context

Beanbag sharing

Time 5–15 minutes

Resources

- PE items (balls, beanbags, and so on)
- Classroom items (pens, and so on)

Key vocabulary

fair	same number
share out	predict/guess
left over	groups
share between two/three	

Teaching activity

In PE lessons and in ordinary classroom activities, try to give further help with sharing and things left over. Explain that the child is going to have some extra practice with sharing because it is such an important thing to learn. And in particular they will be learning about what to do with any left-over things.

Organise the class into, say, six groups. Ask the child to share the basket of beanbags equally between the groups (checking first that there will be some beanbags left over).

? Can you share out these beanbags equally between our six teams of children?

? How many does each group have? Is that fair?

? What shall we do with the beanbags left over?

In this situation, a real-life way of dealing with the left-over beanbags is not to cut them up like you might a cake!

Repeat with pens, and so on, in the classroom.

? If we share out these eleven pens equally between you and your four friends, do you think there might be some left over?

Let the child share them out equally and decide what to do with the left-over pen.

? What have you learned about sharing today?

Spotlight 2

When sharing, can sometimes make equal groups but has no strategies to deal with any left over

Opportunity for: reasoning about numbers

Sharing sweets

Time 10–15 minutes

Resources

- Paper with hands drawn on it, or small world people, or character toys
- Hard sweets to share out

Key vocabulary

fair	same number
share out	predict
left over	groups

Teaching activity

Explain that today we are going to work on sharing fairly. Three children are holding out their hands to have a fair share of five sweets.



? If we share these five sweets out fairly between these three children, do you think there will be some left over?

Help the child to predict first, then to see if they are right by sharing.

? How many sweets do they have each? Is it fair? Are you sure?

? We can't break the sweets so what shall we do with the left-over ones?

Try to draw out some sensible answers about what to do with the sweets.

? If we had five real currant buns and a knife, what could we have done?

If the child is finding this too challenging, you could repeat the activity with three hands, using a different number of items to share.

? Can you think of a number we could share equally between three people?

Try the activity with other numbers of hands.



- ? How many hands have we got this time? If we have three sweets, can they have one sweet each? Why not?**

Let the child try sharing three sweets between four people and see that there are not enough for them to have one each.

- ? What if we have six sweets? Are there enough for one sweet each?**

Let the child do the sharing and talk about the sweets left over.

- ? Can you count some more sweets out of the box, taking enough so that everyone has two each?**

Help the child to work this out in a practical way.

- ? Did we have any sweets left over?**

- ? So what can you tell me about eight sweets and four people?**

- ? If we had one more sweet, so now we have nine, can you predict if they will share out equally between four people with none left over?**

- ? Can you take enough sweets so that each person will have three (or four) each? Can you work out how many more you need to take?**

- ? What have you learned today about equal sharing and coping with left-over things when you share?**

Spotlight 3

When sharing, can sometimes make equal groups but has no strategies to deal with any left over

Opportunity for: exploring a real-life context

Pocket money time

Time 10–15 minutes

Resources

- 1p coins in a purse
- Three other containers
- Pictures of three people or three small world people or character toys

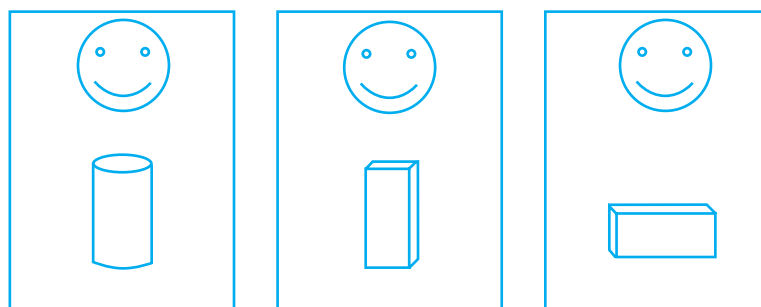
Key vocabulary

- | | |
|-----------|-------------|
| fair | same number |
| share out | predict |
| left over | groups |

Teaching activity

Show the child the 1p coins and explain that you are going to pretend to be Mum/Dad/carer giving three children their pocket money. Show the three containers and representative people and explain that the activity will help the child to learn more about equal sharing and working out what to do with leftovers.

Ask the child to pretend that there are three children and that they are all going to be given an equal amount of pocket money, so that it will be fair.



? If Mum/Dad/carer has three pence, how many pence each can they have? Is there any money left over? Why?

Support the sharing into the three containers.

? What if Mum/Dad/carer has three more 1p coins? How many can they have each? Are there any left over?

Put out some more 1p coins from the purse (make sure there will be some left over).

? Can you give them some more 1p coins so that they all have a fair share?

? How many can they have each?

? Do they all have the same number?

? Are there any 1p coins left over?

If the child is struggling with this, spend time helping them to talk about what their parent/carer might do in this situation. Try to make sure that they can use the key vocabulary above and that they can talk about equal groups with the same number of objects in them.

Explain that any spare money will go back into the purse. Mum/Dad/carer is going to keep it to buy things at the shop. When Mum/Dad/carer gave out the pocket money she counted out the 1p coins carefully so that each person had the same and none were left over.

Talk the child through the three children going to the shop and spending all their money buying comics and crisps. (If you can engage the child at this stage about the actual prices of comics and so on, it might give you insight into how familiar the child is with real money in real situations.)

- ? What did you learn today about sharing equally?**
- ? Where did the left-over 1p coins go?**
- ? What else could Mum/Dad/carer have done with the left-over 1p coins?**

Spotlight 4

When sharing, can sometimes make equal groups but has no strategies to deal with any left over

Opportunity for: developing mathematical language

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Fair shares between two

Time 10–15 minutes

Resources

- 1p coins
- Two purses or containers
- Number cards 2–10 or 2–20 (Resource sheets 1 and 2)
- Pictures of two people or two small world people or two character toys
- 100-square

Key vocabulary

fair	share between two
share out	count in twos
left over	groups
same number	lots of
predict	

Teaching activity

Explain that today the activity will help them to count in twos and to share fairly.

Give the problem a real-life setting, for example, the child and her brother going shopping.

‘Mum/Dad/carer says you must have the same number of 1p coins each. It must be fair. She gets out two 1p coins.’ Put a number 2 card on the desk.

? Can you share these two 1p coins between you and James? Is it fair? Do you have the same number of coins each?

? Can you explain to me what you did?

Give the child two more 1p coins and ask them to share them out fairly. Talk about how many coins they have each and put down a number 4 card.

Ask the child to count out enough pennies so that each child has three 1p coins.

? How many 1p coins are there altogether? Can you count them?

Use the language of ‘two lots of three makes six’ and ‘if you share six equally between two they get three each’. Put down a number 6 card.

? Are there any 1p coins left over? Why not?

Help the child to talk about this using the key vocabulary above.

? Can you put out enough 1p coins so that you and James have four each? How many 1p coins is that altogether?

‘Two lots of four make eight altogether. If you share eight 1p coins equally between two, they have four each and none left over.’

? What number do you think comes next with the number cards? Let’s read them carefully.

‘Two, four, six, eight. Find the numbers on a 100-square.’

? Shall we see if you are right and ten is the next number in the pattern? Were you right?

? So if we share out ten 1p coins equally between two people, are there any left over?

Try to establish that, if you count in twos, all the numbers will always share equally between two people with none left over.

? What if we had nine 1p coins? Could two people have five each? Why not?

Support the child in sharing out the nine 1p coins equally and clarify that there will be one left over. There needs to be one more to have five each. We can't cut the penny in half.

? What is special about sharing the numbers two, four, six, eight, ten between two people?

(Try to get the child to explain that they can be shared out equally between two people with none left over.)



Extend the pattern beyond ten with the whole class, counting in twos to 100 while you point to the numbers on a large 100-square. Ask what is special about these numbers when sharing between two.

? So can you predict if seventy-eight will share out equally between two people with none left over?

Spotlight 5: a learning check

When sharing, can sometimes make equal groups but has no strategies to deal with any left over

Opportunity for: explaining and discussing

.....

Cut it up

Time 10–15 minutes

Resources

- Range of things to share that you haven't used in previous activities, for example, apples, marbles, beads, bricks
- Slices of bread and a blunt knife
- Real or pretend food made with modelling material
- Small world people, puppets or character toys

Check: does the child use key vocabulary?

fair	predict
share out	share between two
left over	groups
same number	

Teaching activity

Remind the child about the equal sharing that they have been doing.

- ? Tell me what you think it means to put things in equal groups. You can use anything on the table to show me what you mean.**

Build on what the child says.

- ? Can you put these nine apples (or other items) into equal groups so that two small world people can share them and both have the same number of apples each?**

Support the child in talking about the sharing.

If the child ignores the apple left over, prompt with questions about what to do with it.

- ? How many apples each do the people have? Is that fair?**

- ? What can you do with this one left over?**

Establish that any left-over apples can be cut up to make it fair.

Move on to sharing three slices of bread equally between two people.

- ? What can you do with the third slice?**

Prompt the child to cut it so that each person has half.

Move on to sharing out bricks.

- ? Can you share out these five/seven/nine bricks equally between two people? Can you predict first what is going to happen?**

- ? What can you do with the left-over brick?**

Establish that you mustn't cut it up! Talk through what two children playing might do when they are sharing out the bricks.

- ? Can you count out enough bricks so that both of us will have three each with none left over?**

Support the counting and the sharing.

? Can you tell me another number, larger than six, which will make equal groups between two people with none left over?

Prompt for eight, ten, and so on.

? What about seven? Can you make a guess first whether seven will share out with none left over?

If the child needs another context for you to assess their understanding, put out three or more small world people and ask the child to share out sweets and explain to you exactly what they are doing.

? Can you explain to me why we sometimes put things in equal groups?

? Can you tell me some of the things that can be cut up to share equally and some that can't?



Explore with all the class the kinds of things that they share at home and what happens to the bits left over. How is a packet of eight sausages shared out in a family of five?

Learning outcomes

By the end of this set of activities, children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- share items fairly;
- predict numbers that will share fairly;
- explain some strategies for dealing with items left over.

Has difficulty with counting reliably in tens from a multiple of ten

Opportunity for: looking for patterns

Resources

- 100-square
- Tens cards (Resource sheet 24)
- Multiples of ten (10–100) written on A5 pieces of paper
- Beanbags

Key vocabulary

ten, twenty, thirty, and so on to a hundred
backwards
forwards

Teaching activity

Time 10–15 minutes

Explain to the child that the activity today is a game called **Whisper and jump**, and it will help them to count in tens.

? What do you think counting in tens means? Can you count in tens?

Show the child the number cards (10, 20, and so on to 100).

? Can you put these cards in order?

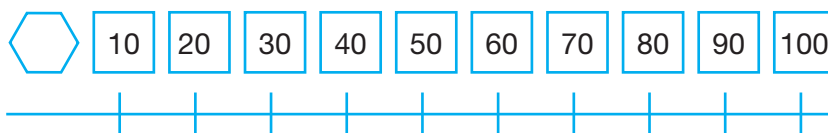
If the child finds this hard, support putting them in order and refer to a 100-square, clearly indicating the vertical line of tens numbers. The child might prefer to order the cards vertically rather than horizontally.

Help the child to count in tens, by rote at first, pointing to the cards.

Suggest that the child tries to do the count with their eyes shut, making a picture of the next ten number in their head.

? Can you start at forty and count on? Which number comes just before one hundred?

If the child needs further support, put a spaced out paper tens number line to 100 on the floor, with something for zero. Put a beanbag by each ten number (but not by zero).



How to play

- The player stands on zero and counts from one to nine in a whisper, but when they get to ten they say it loudly and jump onto the 10 paper. (If you do this in the hall or playground, so that they can shout the tens numbers, it can be good fun and leave a lasting impression!)
- Then they whisper the numbers from eleven to nineteen inclusive and shout and jump to 20, and so on to 100.

? How many jumps and shouts did you do? How many beanbags are there?

Ask the child to shut their eyes while you move some of the tens numbers out of order.

? Can you put the numbers back in the right order?

If the child finds this hard, encourage them to look closely at the numbers to see the patterns of one, two, three, and so on to ten, within the numbers.

Repeat the activity, counting either with cards or the floor number track, but counting back in tens from one hundred to zero. (Counting by whispering all the numbers between the tens is quite hard, so that might be a supporting whole-class task.)

You can say the counting back numbers as a different way, to blast off a rocket – one hundred, ninety, eighty, and so on to zero or ‘blast off’!

? When you are counting back in tens, which number do you say just after twenty? Just before fifty?**? Can you tell me something you learned today?**

Go over counting in tens, forward and back, and the vertical pattern down the edge of a 100-square, and keep this on display.

Count back from one hundred by whispering ninety-nine to ninety-one then shouting ninety, and so on back to zero.

Spotlight 1

Has difficulty with counting reliably in tens from a multiple of ten

Opportunity for: solving practical problems

5 YR X / ÷

Ten trains

Time 10–15 minutes

Resources

- 'Ten trains' of cubes in different colours
- Bead string in tens
- Tens cards (Resource sheet 24)

Key vocabulary

ten, twenty, thirty, and so on to a hundred
backwards
forwards

Teaching activity

Explain that the activity today will help them to be better at counting in tens.

With the 'ten trains', count to a hundred in tens, moving one 'ten train' at a time. 'Ten, twenty, thirty', and so on.

? How many 'ten trains' do we have? How many cubes is that?

'Ten groups of ten make a hundred altogether. Let's count them in tens again.'

? Can you count aloud in tens with your eyes shut and get all the numbers in the right order?

If the child needs further support, count in tens by 'throwing' all ten fingers with each count of ten, twenty, thirty, and so on. Refer to the tens number cards as you do this.

? Can you put the number cards in the right order? Which tens number comes before sixty, after seventy?

Put the different coloured 'ten trains' on the table in a line and ask the child to position the number cards at the right place up to one hundred.



If the child finds this hard, repeat the activity with a bead string in tens, supporting the child putting the tens number cards in the right place.

Ask the child to shut their eyes while you cover or remove one of the tens numbers. Point to the place without the card.

? Which tens number goes here?

? Which one is number thirty? Which two numbers are either side of thirty?

? Are you better at counting in tens now? What do you need to practise a bit more?

Spotlight 2

Has difficulty with counting reliably in tens from a multiple of ten

Opportunity for: working systematically

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Lots of spots

Time 5–10 minutes

Resources

- Sets of five spots (Resource sheet 29)
- Cubes
- Bead string
- Place value (arrow) cards

Key vocabulary

ten, twenty, thirty, and so on to a hundred
backwards
forwards

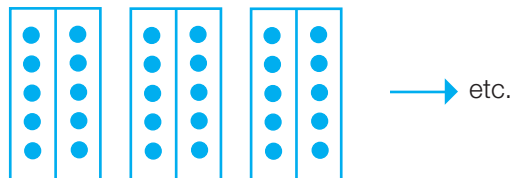
Teaching activity

Explain that we are going to do an activity with spots to help us to count in tens. Show Resource sheet 29 (*Sets of five spots*), cut in half vertically to make one long row of sets of ten (see below).

? Can you count the spots in this first line? (*Ten in two groups of five.*)

If the child finds the counting difficult, or finds it hard to see the spots, make 'five trains' out of cubes, in different colours so that there are two fives in red, two fives in blue, and so on, so that you can construct 'ten trains'. Lay these out in the same arrangement as on Resource sheet 29 (*Sets of five spots*).

Count the spots/cubes in each set of ten to establish that every group of five and five makes ten.



Ask the child to place the right place value card at the end of each set, starting with ten. Try to get the child to predict the next tens number.

? How many spots do you think there are in the next set? Can you guess without counting them?

If the child is finding it hard to work out the next number of tens, look at the 'five and another five' dots and make the link to their fingers.

Remind them how to chant the sequence ten, twenty, thirty and so on. You could use a bead string if the child is finding it hard to get to a hundred.

Work down the sheet, putting the tens place value cards in order, asking the child to predict the next one each time, then to find the right place value card.

? Can you see the pattern of the numbers we are making?

? If the sheet was bigger and had the next sets of dots, how many would there be?

If the child is finding it difficult to remember the pattern of tens, use cube 'ten trains' or a bead string to count in tens again, emphasising that on the sheet the dots are in groups of two fives, which is ten. It might help to point with a finger as the tens of dots are counted.

? Which number comes after sixty? And what is next?

End by asking the child to chant the sequence of counting in tens with their eyes closed. If they can count forwards to a hundred, see if they can start at one hundred and count in tens backwards.

? What did you like doing best today?



Chant in tens to two hundred or beyond with the whole class. Remember to chant backwards as well as forwards.

Spotlight 3

Has difficulty with counting reliably in tens from a multiple of ten

Opportunity for: exploring a real-life context

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Hidden tens

Time 10–15 minutes

Resources

- Place value (arrow) cards
- 10p coins and rewards
- Small world person, character toy

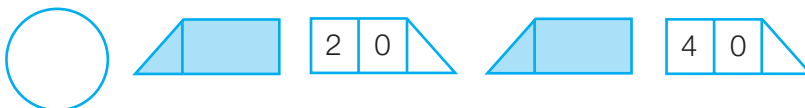
Key vocabulary

ten, twenty, thirty, and so on to a hundred

Teaching activity

Explain that this is an activity to help with counting in tens up to a hundred and back again, but today some of the numbers will be hidden.

Ask the child to put out the place value cards, in order from ten, as a line across the table. Help them to count along it and back again saying the tens numbers. Then turn some of the cards over. You can use a toy to jump along the line, or let children use their fingers as legs to count along the line. (You might want to put a plastic circle for zero.)



? Can you work out which card is thirty?

? Can you use the toy to count on from thirty and stop when you get to seventy?

Repeat with a variety of numbers. For example: 'Stand on fifty and jump to ninety. Turn the card over and see if you are right.'

If this proves too challenging, let the child turn the numbers over one at a time, guessing which number is hidden. Repeat this as a game, giving some kind of reward for correct numbers, for example, a plastic ten pence coin (if they win ten of these they can have a biscuit).

Work towards all the numbers being out of sight and the child jumping along the line forwards and backwards saying the sequence of numbers.

? Can you stand on the number that is ten more than seventy/ten less than fifty/twenty more than twenty?

Move on to using 10p coins. Count out the coins until you have a pound. Ask the child to count out fifty pence, and so on.

? What is ten pence more than fifty pence?

Now hide the coins and ask the child to shut their eyes and make pictures of the coins in their head.

? What is ten pence more than sixty pence/twenty pence less than fifty pence?

Return to the number line with a few of the numbers showing.

? Can you tell me any of the hidden tens numbers?

? Are you getting better at counting in tens?

Spotlight 4

Has difficulty with counting reliably in tens from a multiple of ten

Opportunity for: using and applying mathematical knowledge

Tens race

Time 5–15 minutes

Resources

- A few sets of tens number cards (Resource sheet 24)
- Timer or stop-watch, or plastic tocker timers

Key vocabulary

ten, twenty, thirty, and so on to a hundred
backwards
forwards

Teaching activity

'Today you are going to play **Tens race** to help you to get better at counting in tens.'

How to play

1. Ask the child to put one pack of tens cards in order. These are left on the desk for the child to refer to as they play the game. Use your knowledge of the child to decide how many packs of cards to play with and how much time to allow them.

Time the activity so that if the child has struggled to put out the first set of cards, you can time them again with the stop-watch as they put out the next set and challenge them to beat their own time.

2. Gradually make the timing more challenging.

? Can you put these two packs of tens cards in order in one minute?

3. When the child is confident with this, put away the pack of cards left on the desk so they have to remember the order.

? Can you put this set of tens cards in order before the timer stops?

? Can you put the cards in order starting with the 100 card and counting backwards?

? What did you enjoy about the game today?

? Are you getting better at counting in tens?

? What would you like more help with?



Challenge the class to make up their own card race games, which they can time and try to get quicker and quicker. Some might be able to race to put cards in order, counting in threes or fours or sevens. Remember to challenge them to count backwards as well as forwards.

Spotlight 5: a learning check

Has difficulty with counting reliably in tens from a multiple of ten

Opportunity for: explaining and discussing

Eyes closed

Time 5–15 minutes

Resources

- Large blank 100-square
- Two sets of tens cards (Resource sheet 24)
- Real 2p coin with 'b' on one side and 'f' on the other
- Counting stick, metre stick, scales
- Reward

Check: does the child use key vocabulary?

ten, twenty, thirty, and so on to a hundred
backwards
forwards

Teaching activity

'The task today is a game, **Eyes closed**, where you are going to shut your eyes and do some counting in tens. We are going to toss the coin. If it lands so we can see "b" for backwards, you should count back all the way to zero, and if it lands with "f" for forwards, you should count forwards all the way to a hundred.'

How to play

1. First, ask the child to put tens cards from 10 up to and including 100 in the right place on the 100-square, and explain that they can look at that if they get stuck.
2. With a shuffled set of tens cards, ask the child to take one, for example, twenty. Then toss the coin. If it lands on 'f', the child counts from twenty all the way to a hundred, but with their eyes shut. If it lands on a 'b' they count from twenty back to zero.

3. If they are correct, they win a reward. (The card is not put back in the pile you are using.)

If this proves too challenging, or the child gets confused, let them look up at the 100-square at any time, then go back to having their eyes closed.

4. Repeat the card game a few times with different cards.

? Can you tell me which tens number comes after seventy?

? Do you find it easier to count forwards or backwards?

? If you are counting forwards, which number comes just after forty? Explain to me how you knew that.

? Can you explain to me why counting in tens is so important?

If the child is unsure about this, remind them of 10p coins and look at a metre stick and a dial on metric scales where the tens are marked boldly.

Using an unmarked counting stick, ask the child to count along it in tens, silently, pointing at the right place on the stick. After they have understood what you mean, explain that you are going to stop them sometimes and ask them the number that is in their head. This should be the number that they are pointing to.



Extend the silent counting using a counting stick during whole-class mental mathematics times.

Learning outcomes

By the end of this set of activities, children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- count in tens forwards and backwards from any tens number up to a hundred;
- say a number that is either side of a tens number;
- be able to recognise a simple number pattern and talk about it.

When halving, makes two unequal groups or splits a single object unequally

Opportunity for: exploring a real-life context

Resources

- Cubes or sweets, chocolate bar, modelling material buns or sausages
- Apples and a blunt knife
- Biscuits
- Piece of string and scissors
- Jug of water and two beakers

Key vocabulary

half	fair
halve it	equal
half of	same amount

Teaching activity

Time 10–15 minutes

Explain to the child that the activity today will help them to find out more about halving things.

Put some of the things to halve on the table, including the cubes.

? What do you think 'half' means?

Talk about phrases with half in them, for example half an hour, half an apple, a half moon, a half-back in sports, half-baked, half a mo, too clever by half.

? Can you give me half of these four cubes and you have the other half? Is it fair?

Repeat with different numbers.

If the child finds halving numbers of objects hard, it might be that halving quantities will be easier and will help with understanding the concept of what 'half' means.

Put out some of the items for halving, for example a sausage or a bun, and ask the child to halve them.

? Can you explain to me why you cut the sausage there?

? Can you half-fill this beaker with water? Why do you think that is half-full?

? Can you cut this string in half? Is that fair? Are the two pieces the same size?

(They have to be, to be called halves! If something is 'halved' but ends up rather different sizes, for example, if someone is trying to break a crumbly biscuit in half, you could talk about it if the child seems to be able to cope with this. You can simply say how hard it is to divide a biscuit into accurate halves, so we can just say it has been divided into two pieces.)

? How many people can share an apple if they have half each?

If the child seems not to be understanding halving quantities, you will need to use some more examples, perhaps with modelling material made into a range of pretend food, but also try to halve real things like apples, cakes, chocolate bars, and so on.

If you have time, try halving some larger numbers.

? Can you halve these twelve sweets so that I have half and you have half? Is it fair? How many groups did you split the sweets into?

Challenge children with the numbers you use. If they can halve twelve, try sixteen, twenty, and so on.

If the child cannot halve numbers above twelve, you might need to write down the halves that they can do and work at halving groups of cubes in ordinary class time and recording what they do. Encourage the child to learn the results of their halving by heart.

? What did you learn today about halves?

Spotlight 1

When halving, makes two unequal groups or splits a single object unequally

Opportunity for: solving practical problems

Half a cake

Time 10–20 minutes

Resources

- Modelling material, cake or pizza, muffins, apples, banana
- Blunt knife
- Balance scales

Key vocabulary

half	equal
halve it	heavy
half of	light
fair	same amount

Teaching activity

Explain that you are going to do an activity of halving a cake and using the balance scales to check that each bit weighs the same.

First establish that the child understands balance scales.

? What will happen if I put this whole cake on this side of the balance scales?

If the child is unsure of what will happen, let them experiment for a while, putting heavy and light things on the scales to see what happens.

Establish that the bucket containing the heaviest thing goes down and the bucket containing the lightest thing goes up.

? Can you cut the cake in half and put the parts in the buckets so that the buckets are level?

? If the buckets are level, what does it tell you about the halves you made?

Work with the child, taking bits of cake from one side to the other until they more or less balance. Explain that halves should be exactly equal, but sometimes it can be hard to get it exact.

? What can you tell me about our two bits of cake now?

? If you have that bit and I have this bit, is it fair?

? If we cut something in half, how many pieces do we have?

Use another cake, or a piece of fruit, and cut it unequally so that one bit is much larger.

? Are these two halves?

? Why not?

? What did you learn today about halves?



Ask for some suggestions of things we often halve, for example sausages, a medium pizza when we share between two, rolls we cut in half to put cheese inside, tomatoes, apples, bars of chocolate, and so on.

Spotlight 2

When halving, makes two unequal groups or splits a single object unequally

Opportunity for: exploring a real-life problem

.....

Half the money

Time 10–15 minutes

Resources

- Range of coins that the child is familiar with, for example 1p and 10p coins
- Two small character toys or puppets
- Biscuits

Key vocabulary

half	fair
halve it	equal
half of	same amount

Teaching activity

Explain that today they are going to practise sharing money between two people, so that they have half each.

Put the two character toys or puppets on the table with coins to suit the child, for example six 1p and two 10p coins.

? Can you give half of the money to this person and half to that person?

Establish that when we halve money, we don't mean we cut it in half as we would with an apple!

If the child finds the money difficult, just use pennies and help with the counting.

? How many pennies/how much does this person have?

? Is that fair? Do they have the same amount each? How do you know?

If the child is finding this too challenging, give them a biscuit or a piece of paper and ask them to cut it in half, so that you both have half.

? How much of the biscuit have you got?

? How much have I got?

? When you split something in half, how many pieces do you have?

If you have time, return to the money and try once more to halve a small amount.

? Tell me what you learned today.



Halve some money amounts. Ask the children to tell you an amount of money that would be hard to share between two people. Get them to consider what you might do in real life if you couldn't exactly halve some money.

Spotlight 3

When halving, makes two unequal groups or splits a single object unequally

Opportunity for: making decisions

6 YR X / ÷

Fold it

Time 10–20 minutes

Resources

- Various shapes of paper (all symmetrical)
- Scissors

Key vocabulary

half	fair
halve it	equal
half of	same amount

Teaching activity

‘Today we are going to do some folding of shapes. We are going to fold them into halves.’

Put the paper shapes on the table and ask the child to choose one to fold in half.



If the child finds this hard, focus on just one of the shapes, help with the folding and talk through how one half is the same size and shape as the other half.

It might help if the child cuts along the fold and you can place one half on top of the other half to demonstrate that they are both the same size and shape.

? Why did you fold it just there?

? Is each half the same size, or is one bit larger than the other?

? How many pieces have you cut the shape into?

If you have time, make a difficult shape to halve, for example a roughly triangular torn piece of paper.

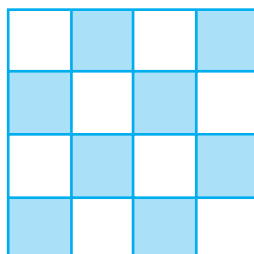
? Do you think it would be hard or easy to fold this in half?

? Is it halved, or is it just divided into two parts?

? What did you enjoy doing today? What did you do well?



Ask the children how many ways they can colour half of a four-by-four grid of squares.



Spotlight 4

When halving, makes two unequal groups or splits a single object unequally

Opportunity for: solving practical problems

How many halving

Time 10–15 minutes

Resources

- Counters, cubes or sweets
- Dominoes

Key vocabulary

half	equal
halve it	same amount
half of	how many
fair	

Teaching activity

‘Today we are going to do lots of halving of numbers. Sometimes you have to share sweets or other things by dividing in half, so it is an important thing to learn.’

? Can you count out some of these cubes (or counters or sweets) and then halve that number of cubes?

? How many did you count out?

? What is half of that number?

? Is it fair? Are there two equal groups?

If the child has counted out too many, go back to halving just four or six cubes until they feel more confident.

Make the point that they are splitting the number of cubes into two equal groups so that it is fair.

Take ten cubes (or counters or sweets) and divide them into two piles of five. Make sure the child knows that there are ten items in total.

? How many are in this pile?

? How many are here? So are the piles equal in number?

? Are there the same number there as here?

? So what is half of ten?

If the child is still struggling, use dominoes to show halves, for example half of six is three on each side.

If the child is coping with smaller numbers, move on to numbers above twelve.

? What is the largest number you think you can halve?

If you have time, ask the child to double one of the halves. For example, if they halve ten and get five, ask them to double five so that they get back to ten.



? What is the largest number you think you can halve?

? What happens if you halve a number, then double the answer?

Spotlight 5: a learning check

When halving, makes two unequal groups or splits a single object unequally

Opportunity for: explaining and discussing

Happy halving

Time 10–15 minutes

Resources

- Counters or buttons
- Ribbon or rope
- Pizza made of paper or modelling material
- Beakers and jug of water

Check: does the child use key vocabulary?

half	fair
halve it	equal
half of	same amount

Teaching activity

Explain that today we are going to be finding halves of things. Ask the child to look at all the things on the table.

? Can you choose something to show me how much you know about finding halves?

Build on what the child chooses.

? So how did you know that was a half?

? Can you find the length of rope/ribbon that is half as long as this piece?

? Can you tell me why you chose that bit?

? Can you put some water in this beaker so that the beaker is half-full?

? How did you know that was about half-full?

If the child finds it hard to explain about the water in the beakers, tip some out of one into the other so that they hold different amounts.

? Have I shared this water out fairly? Do you have the same amount as me?

? If you and I were going to share this pizza equally, can you show me how much I could have?

? How much would you have?

? Can you give me half of this pile of buttons?

? Are you sure you have halved that group? I wonder how you know.

? Can you explain to me what we mean by halving something or finding half of a number?

 **? Can you draw half of something? Half a dinosaur? Half a rainbow? Half a space ship? Half a smiley face? How could using a mirror help you?**

Learning outcomes

By the end of this set of activities, children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- halve a number of objects by making two equal groups;
- find half of a quantity in a range of appropriate ways;
- talk about halves using relevant vocabulary.

Still counts in ones to find how many there are in a collection of equal groups; does not understand vocabulary, for example, 'groups of', 'multiplied by'

Opportunity for: developing mathematical language

Resources

- Cubes or counters
- Number line
- 2p and 10p coins
- Base 10 apparatus
- Purse

Key vocabulary

array	how much?
arrangement	count
count in twos	altogether
columns of ten	

Teaching activity

Time 15–20 minutes

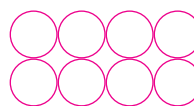
Explain that this activity will help the child to count in twos and in tens, which will make counting numbers of objects easier.

Take a collection of eight cubes/counters of the same colour. Ask the child to arrange the eight cubes/counters in a way which will make them easy to count. Repeat this several times with collections of eight, asking the child to make a different arrangement each time. Leave each collection as arranged by the child.

? How many cubes (or counters) are there in each collection? How do you know?

Ask the child to count each collection in turn.

If this arrangement has not been made, make it using another eight counters.



? Which arrangement did you find easiest to count and why?

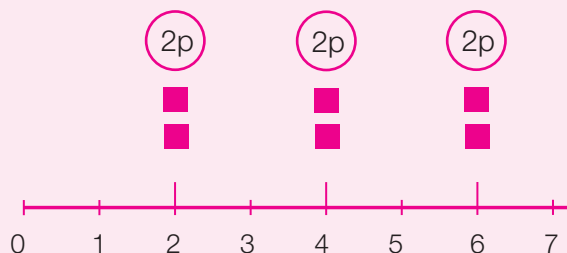
If the child is not comfortable counting in twos, use the 2×4 arrangement (see above) to practise counting in twos, adding on column by column. Encourage the child to see the counters as arranged in four groups of two.

Explain that we are going to work more on counting in twos, as this is an easy way to count.

Give the child a purse containing 2p coins. Ask the child to work out how much money they have by counting in twos.

Repeat this several times with different numbers of 2p coins in the purse.

If the child has difficulty counting the amounts made up from the 2p coins, use pairs of coloured counters/cubes to show twos along a number line up to 20.



Support the child in counting in twos up to 12, 16, 20.

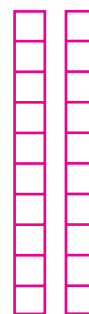
Place 2p coins above the pairs of counters/cubes, and repeat the activity, emphasising the coins rather than the counters/cubes.

Set up a 10×2 array of counters/cubes of the same colour.

? How many cubes are there in each column?

? How many cubes are there altogether?

If the child can tell you that there are ten in each column and twenty altogether, then add another column of ten and ask the child to tell you how many there are now. Keep adding columns of ten, ensuring that the child is still counting in tens.



If the child cannot recognise that the array shows columns of ten, count out twenty unit cubes (Base 10). Show this in two groups of ten. Ask the child to match the 10-stick to the correct number of unit cubes. Now ask the child to match the second group of ten with another 10-stick. Ask the child to show thirty. Continue as necessary.

Give the child a bag of 10p coins. Ask the child to work out how much money they have by counting in tens.

? What did you learn today about making counting easier?

Spotlight 1

Still counts in ones to find how many there are in a collection of equal groups; does not understand vocabulary, for example, ‘groups of’, ‘multiplied by’

Opportunity for: being systematic

.....

Grab! **Time** 15–20 minutes

Resources

- Cubes
- Collection of small objects, e.g. counters

Key vocabulary

- | | |
|---------------|------------|
| count in twos | count |
| count in tens | altogether |
| how many? | |

Teaching activity

‘Today we are going to do some counting in twos and tens so that we can count these counters quickly. Then we are going to check our count to see if we are right.’

Place a large number of small objects in a container and ask the child to grasp two handfuls, one in each hand. (Cubes keep numbers smaller, but also try the activity with tiny items to check how far the child can count.) The aim of the game is to count the objects as quickly and accurately as possible. Direct the child to count one handful in twos or tens, while you count the other in ones. Who can count the handful of objects more quickly?

If the child cannot manage the above activity, sort the objects into groups of two or ten and then count together.

Repeat with different handfuls, making the quantities smaller if necessary.

? Do you think it is quicker to count in ones or to count in twos?

‘Let’s see if we can put these counters into twos, then tens. Then we will be able to check if the count is right.’

Repeat with smaller objects.

Note at what point the child’s counting breaks down, in order to extend it over the next few weeks.

? Do you think you are getting better at counting? How do you know?

? What do you think is the quickest way of counting that we have used?



‘Race your friend to count a handful of small counters.’

Spotlight 2

Still counts in ones to find how many there are in a collection of equal groups; does not understand vocabulary, for example 'groups of', 'multiplied by'

Opportunity for: investigating numbers and predicting

.....

On the calculator

Time 15–20 minutes

Resources

- Calculator
- Small cubes
- 2p coins and 10p coins
- 100-square

Key vocabulary

- | | |
|---------------|---------|
| count in twos | count |
| count in tens | predict |

Teaching activity

'Today we are going to find out something very special that we can do on the calculator: we can make it count in twos.'

Show the child how to key in a constant function on the calculator. On most simple calculators this is

usually 2 + = = = = but sometimes there is minor variation.

'Now you press the equals key and look at what is happening.'

? What do you think will be the next number? And the next?

If the child does not follow, go back to zero again and repeat. Then ask the child to key in from the start and say the numbers out loud a few times so that they will be able to do this on their own in class to practise.

'Let's start from zero again and you press equals three times.'

? What number will show on the screen after three presses?

? Find the number pattern on the 100-square and predict the next two numbers.

If the child still seems unsure, support the counting with cubes and 2p coins.

Repeat with 1 0 + = = = =

Support this counting with 10p coins and find the tens pattern on the 100-square.

Encourage the child to show friends in class what they can do and to repeat the activity in class and at home until they can count securely to fifty then a hundred and beyond.

? What did you enjoy in today's session?

Spotlight 3

Still counts in ones to find how many there are in a collection of equal groups; does not understand vocabulary, for example, 'groups of', 'multiplied by'

Opportunity for: reasoning with numbers

.....

Tens and twos

Time 10–15 minutes

Resources

- Counting stick
- Square sticky notes
- Number line

Key vocabulary

- | | |
|---------------|------------------|
| count in twos | multiples of two |
| count in tens | multiples of ten |

Teaching activity

Write multiples of two on sticky notes and place these on a counting stick, with some out of order.

'We're going to do some counting today in twos and tens on the counting stick, but I think I've got some of the numbers out of order.'

? Put the numbers in order and count in twos along the counting stick.

If the child is finding this too hard, support them to put the sticky notes in order using a number line to help.

Support the child counting in twos to ten, then twenty.

Ask the child to close their eyes while you take away some of the sticky notes.

? Which number goes in this gap?

Repeat until the child is more confident, and encourage them to do this back in class and at home. Repeat with sticky notes and multiples of ten.

? What did you like doing best today?

? Do you think you are getting better at counting in twos and tens?

Spotlight 4

Still counts in ones to find how many there are in a collection of equal groups; does not understand vocabulary, for example, ‘groups of’, ‘multiplied by’

Opportunity for: making connections

.....

All the fives

Time 10–20 minutes

Resources

- Cubes in ‘five trains’
- Sets of five spots (Resource sheet 29)
- Calculator
- Number line
- Place value (arrow) cards

Key vocabulary

- | | |
|----------------|----------------------|
| count in fives | how many altogether? |
| columns of ten | tally |

Teaching activity

‘We’re going to do some counting in fives using these “five trains” and the calculator.’

? Can you remember how to make the calculator count in tens? Show me.

? Now make the calculator count in fives.

Show the child how to key in

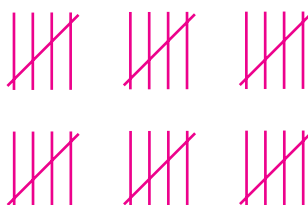
? How far can you count in fives? Look at the screen to help you.

Then ask the child to take a handful of the ‘five trains’ and to count these in fives.

? How many lots of five are there? Can you count along the number line to that number in jumps of five?

? Can you remember how to tally in fives?

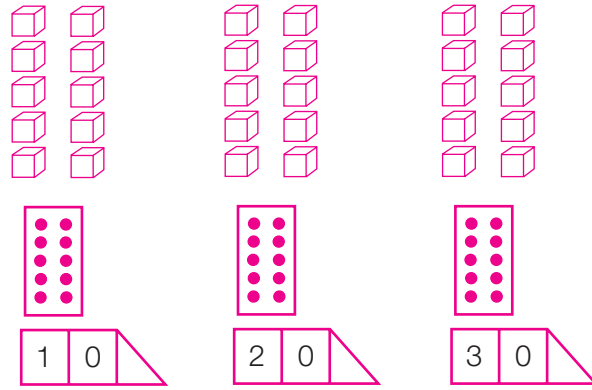
Support the child counting the number of cubes they have altogether in their handful of ‘five trains’.



‘So we have thirty cubes altogether, and they make six “trains” of five cubes. Now let’s put the “five trains” together to make “ten trains”.’

? How many ‘ten trains’ do you think we can make?

Support this counting of fives and then tens using Resource sheet 29 (Sets of five spots).



? What can you tell me about counting in fives and counting in tens?

Spotlight 5: a learning check

Still counts in ones to find how many there are in a collection of equal groups; does not understand vocabulary, for example, 'groups of', 'multiplied by'

Opportunity for: explaining and discussing

.....

Race to fifty pence

Time 15–20 minutes

Resources

- 2p and 10p coins (possibly 1p coins)
- Money counting boards (one for each player or pair)
- At least two children
- Dice 1–4 or a spinner marked 1–4
(Resource sheet 13) (possibly 1–6 dice)
- Timer

Check: does the child use key vocabulary?

- | | |
|---------------|------------|
| count in twos | count |
| count in tens | altogether |
| how much? | column |

Teaching activity

This game can be played as a race, or by pairs cooperating, maybe with a timer to see how long they take.

'Today we are going to play a game called **Race to fifty pence**. It will help you to count in twos and in tens.'

How to play

1. Each pair or player takes turns at spinning the spinner. They then win that number of 2p coins and put those in the 2p column.

Count these coins in twos. Do you have enough 2p coins to exchange for a 10p coin?

2. When a player or pair has five 2p coins, they must explain how they are going to exchange that ten pence for a 10p coin and put that in the ten pence column.
3. Play until someone reaches fifty pence.

Variations

- Set the timer and see how much money one pair can collect in five minutes.
- Play with 1p coins and 10p coins and bigger numbers on the dice or spinner, for example a 1–6 dice and race to collect £1.

Learning outcomes

By the end of this set of activities, children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- count in twos, fives and tens.

Does not link counting up in equal steps to the operation of multiplication; does not use the vocabulary associated with multiplication

Opportunity for: reasoning about numbers

Resources

- Pendulum (or metronome or string with bob attached)
- Sets of 2- and 10-dot cards (Resource sheet 30)
- *Swing cards* (Resource sheet 31)
- Mini whiteboard

Key vocabulary

- | | |
|---------------|--------------|
| multiplied by | steps of two |
| altogether | steps of ten |

Teaching activity

Time 20 minutes

Explain to the child that this activity is going to help them to learn more about multiplying.

Show the pendulum to the child and model counting in twos. Repeat counting with the child, stopping when twelve is reached.

? What is the size of the steps we are counting in?

If the child recognises steps of two, confirm the answer and move on.

Place on the table a digit card with a large number 2. Next to this place a pack of 'swing' cards. Each card is labelled with a different number of swings. Place the '3 swings' card on the top of the pack.



Model this by counting two for each swing of the pendulum for three swings ('two and two and two').

? How many have I counted altogether?

'Repeat this with me: two and two and two. That is two multiplied by three.'

Now place the '4 swings' card on the top of the pack.

? What's the size of step we are counting in? (2)

? Look at the swing card. How many times are you going to swing the pendulum? (4)

'Now we are doing four swings: two and two and two and two. So that is two multiplied by four.'



Repeat using other swing cards.

If the child cannot relate cards to pendulum swings, initially reinforce the step size by modelling the counting in twos (using the 2-dot cards) while the child counts the swings.

Say, 'I counted in twos and multiplied by (number of swings).'

? What is the step size shown on each card? (2)

? How many steps or swings will we count?

Repeat the activity, asking the child to choose different swing cards to illustrate this.

Use a whiteboard to show how the multiplications can be recorded, relating each to the pendulum model, reinforcing the language of step size (2) and swings.

$2 \times 1 = 2$	• •		
$2 \times 2 = 4$	• •	• •	
$2 \times 3 = 6$	• •	• •	• •

If the child cannot see the relationship between written multiplication and dot representation, use the 2-dot cards to make a careful match between the step size and the multiplication, for example:

• • • • • • '2 multiplied by 3', which is written as 2×3 ,

and demonstrate the recording to match.

Give the child other sets of 2-dot cards, and ask them to write a multiplication to match each set.

If the child is confident with the representation of step size and number of steps, repeat the activity using the pendulum and 10-dot cards.

? What shall we write down to remember for next time?

Spotlight 1

Does not link counting up in equal steps to the operation of multiplication; does not use the vocabulary associated with multiplication

Opportunity for: recognising relationships

2 Y2 $\times$ / $\div$

Toy jumps

Time 15–20 minutes

Resources

- Digit card for the number 2 (Resource sheet 1)
- Sets of 2- and 10-dot cards (Resource sheet 30)
- Number line
- Small character toy
- Mini whiteboard

Key vocabulary

- | | |
|---------------|------------|
| multiplied by | altogether |
| steps of two | jump |
| steps of ten | |

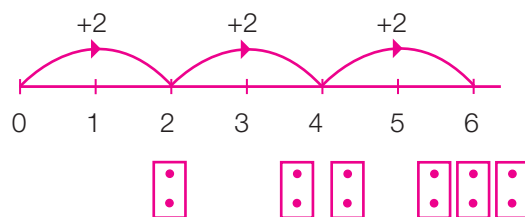
Teaching activity

Explain to the child that this activity is going to help them to learn more about multiplying by two.

Put a digit card showing 2 on the table and ask the child to put three of the 2-dot cards together.

? Look at the three sets of dots. How many dots are there altogether?

Support the child counting up in twos: two, four, six ...



Now use a small character toy to jump along a number line in jumps of two, counting in twos up to 6.

? How many jumps did the toy make?

? How far did the toy jump altogether?

Ask the child to jump the toy along the number line in jumps of two, stopping at 12. As the child moves the toy, reinforce the associated vocabulary by saying 'two multiplied by one', 'two multiplied by two', etc. to mirror the toy's movement along the number line.

? How many jumps did the toy make?

Write the associated multiplication on a whiteboard, i.e. 2×6 . 'We say this as "two multiplied by six".' Ask the child to show the toy jumping along the number line again.

If the child cannot do this, show them four jumps of two and make the connection to two multiplied by four, writing the associated multiplication as $2 \times 4 = 8$.

Now or another time you might want to repeat this with 10-dot cards.

? Let's write down the words we said today when we had six lots of two dots.

two multiplied by six

Spotlight 2

Does not link counting up in equal steps to the operation of multiplication; does not use the vocabulary associated with multiplication

Opportunity for: exploring mathematical language

Interlocking bricks

Time 15–20 minutes

Resources

- Set of 2- and 10-dot cards (Resource sheet 30)
- Interlocking construction bricks with pairs of prongs (e.g. Lego, Duplo)

Key vocabulary

multiplied by	steps of two
altogether	steps of ten

Teaching activity

Explain to the child that this activity is going to help them to learn more about multiplying by two.

Using a brick with pairs of prongs, count in twos, moving along the brick to correspond with the counting.

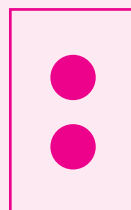
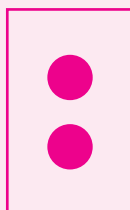
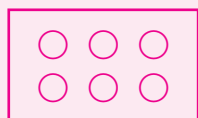


Show the number of prongs altogether on the brick by counting up in twos, for example, 'two, four, six, so there are six prongs altogether'. With the child, chant 'I've got two, one times', 'I've got two, two times', 'I've got two, three times'. Say that this is 'two multiplied by three'.

2 multiplied by 3

$$2 \times 3$$

If the child has difficulty with counting in twos across the brick, show the connection between the brick pattern and the 2-dot cards used in the opening teaching sequence. For example:



two multiplied by three

$$2 \times 3$$

Select another brick (or pair of bricks, placed end on), and repeat the activity. Explain that the child has to count across the brick(s), in twos, to find out how many prongs there are altogether. Support the child in counting in twos to find the total number of prongs on the brick(s).

? How many twos are there on the brick(s)?

? How many prongs are there altogether on the brick(s)?

? Tell me how we said this as a multiplication. Show me how to write this.

Repeat the activity with different sizes of brick or combinations of bricks.

? Tell me what you know about how we say and write multiplications.

Spotlight 3

Does not link counting up in equal steps to the operation of multiplication; does not use the vocabulary associated with multiplication

Opportunity for: exploring patterns

.....

Tapping pattern

Time 10–15 minutes

Resources

- Small tub of paint
- Paper

Key vocabulary

- | | |
|---------------|--------------|
| multiplied by | steps of two |
| altogether | steps of ten |

Teaching activity

Explain to the child that this activity is going to help them to learn more about multiplying.

Help the child to understand two multiplied by three by tapping two fingers on the edge of the table, three times. Repeat this and explain that you are counting in twos, ‘two, four, six’.

? How many fingers did I tap each time?

? How many times did I tap my fingers?

? How many fingerprints did I leave on the table?

Support the child in recording the tapping pattern as $2 \times 3 = 6$.

If the child has difficulty visualising the ‘fingerprints’, use paint on two fingers to leave an image on paper as the finger tapping is done.

Repeat the activity varying the number of two-finger taps.

If the child is confident with two as a step size, repeat the activity using both hands to show a step size of ten.

? Tell me something you liked doing today.

Spotlight 4

Does not link counting up in equal steps to the operation of multiplication; does not use the vocabulary associated with multiplication

Opportunity for: solving real-life problems

.....

Socks

Time 10–20 minutes

Resources

- Cubes or socks

Key vocabulary

multiplied by	altogether
steps of two	how many?
pairs	

Teaching activity

Explain to the child that this activity is going to help them to learn more about multiplying.

Tell a story explaining that Joe's mum wants to buy him enough pairs of socks so that he has a different pair for each day in the school week. She wants to know how many socks she will need to buy altogether.

? How many socks are there in a pair?

? How many days are we at school in a week?

Ask the child to draw a representation of this and find the total number of socks which Joe's mother needs to buy. Discuss the child's representation, ensuring that the drawing clearly distinguishes between pairs of socks and the number of pairs needed.

? What multiplication can you write to match this story?

Support the child in writing:

2 multiplied by 5

2×5

Ask the child to think of other items that usually come in twos (or tens). Give a multiplication statement and ask the child to think of a story to match the statement, for example, 10×6 .

'Seeds come in packets of ten, and I want to buy six packs. How many seeds will I have altogether?'

If the child cannot create his/her own story, provide a suitable story. Support the child in representing the story with a drawing, before linking in the associated multiplication calculation.

? I wonder if you can you write a number sentence for two multiplied by six? Well done!

Spotlight 5: a learning check

Does not link counting up in equal steps to the operation of multiplication; does not use the vocabulary associated with multiplication

Opportunity for: explaining and discussing

Match it

Time 15–20 minutes

Resources

- *Match it cards and game board*
(Resource sheet 32)
- At least one other child
- Timer

Check: does the child use key vocabulary?

- | | |
|---------------|--------------------|
| multiplied by | steps/jumps of two |
| altogether | groups of two |

Teaching activity

Cut out the cards from the board on Resource sheet 32 (*Match it cards and game boards*), so that each card is separate. You can cut the board into three or keep the sections together. This is a cooperative game for children to play in pairs.

‘Today we are playing a game called **Match it**, and you will have to think hard about all the work we have been doing on multiplication.’

How to play

1. Lay all the cards face up on the table near the board.
2. Everyone looks carefully at the three sections on the board and counts how many items there are in each section.
3. Players then pick up a card that they think they can match with a picture. For example, ‘I’ve got two, three times’ goes with the number line jumps of two, so that card is put next to that part of the board.

? Why do you think that card goes with that picture?

4. Play continues until every card is next to a picture on the board.

Variations

- Set a timer and see how long it takes them to do it. Then they have to do it again and try to beat their personal best.
- Ask the players to draw three different pictures and make their own cards. Then see how long a friend takes to place all the new cards.
- The players could make some cards that don’t go with the pictures and try to trick their friends.

Learning outcomes

By the end of this set of activities, children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- link counting in equal steps to multiplication by the number of steps;
- record a simple multiplication using the correct vocabulary to describe the symbols recorded.

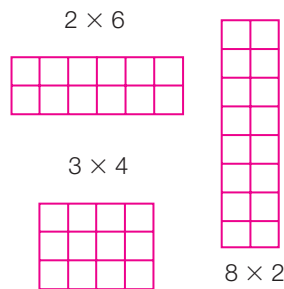
Notes

Does not focus on 'rows of' or 'columns of', but only sees an array as a collection of ones.

Opportunity for: reasoning about numbers

Resources

- Counters
- 2 cm squared paper
- Paper grids made from 2 cm squared paper for the following arrays:
 1×12 , 3×4 , 2×6 , 2×8 , 1×16 , 4×4 ,
 8×2 (others created as required)
- Mini whiteboards and pens



Key vocabulary

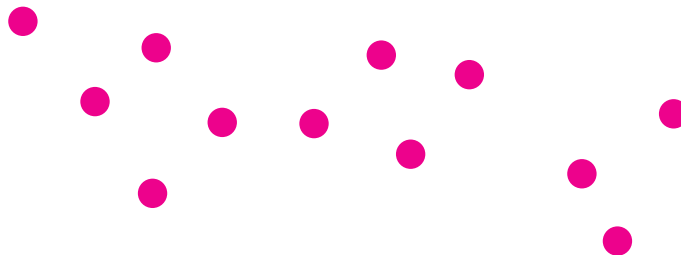
rows	count how many
columns	multiplied by
array	pattern
arranged	groups of

Teaching activity

Time 15–20 minutes

Explain to the child that this activity is going to help them to learn about using rows and columns to help them count more easily.

Spread twelve counters, all the same colour, on the table so that they are not arranged as an array (and cover them so the child does not count them), for example:



Reveal the counters and then hide them again.

? Roughly how many counters do you think there are?

? How did you decide?

Invite the child to check the actual number of counters.

If the child does not arrange the counters in a row to assist their checking, encourage them to do this on a 1×12 array (i.e. 1 row $\times$ 12 columns, or $1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 12$, or 12 groups of 1, or 1 multiplied by 12) squared paper strip, using the vocabulary of 'row' to describe the arrangement.

If the child does arrange the counters in a row, move them onto a paper grid.



Establish that there are twelve counters and that counting in ones is not the quickest way to count.

? Are there other ways in which you could arrange the counters to make the counting quicker and more efficient?

If the child does not choose to arrange the counters in a rectangular array, provide paper grids for other possible arrays, such as 2×6 (2 rows by 6 columns, or $2 + 2 + 2 + 2 + 2 + 2 = 12$) and 3×4 (3 rows by 4 columns, or $3 + 3 + 3 + 3 = 12$), and support the child in moving the counters onto each of the paper grids to establish that other rectangular arrays are possible using twelve counters.

Guide the child to count in twos up to twelve (for 2×6 grid) and/or in threes up to twelve (for 3×4 grid) to illustrate the support that arranging in rectangular arrays can provide.

Repeat the activity with sixteen counters and suitable paper grids to support the process.

Make two or three arrays up to 6×6 using counters of a single colour in each array. Use the corner of a piece of paper to mask all but the far left column and the top row of each array, for example:



? How many rows are under the paper? How many rows altogether?

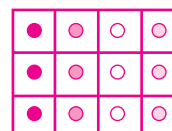
? How many columns are under the paper? How many columns altogether?

Repeat for two or three arrays.

If the child has difficulty with larger arrays, repeat the activity with arrays for smaller numbers.

Within arrays, different colours of counters are now going to be used to highlight rows and columns and to emphasise ways of describing arrays.

Arrange a 3×4 rectangular array on a 3×4 paper grid, using four different colours to indicate the four columns.



? How many counters are there altogether?

? How many are there of each colour?

? How many threes are there?

Support the child to count in threes up to twelve as the columns of the array are indicated.

? Can you record on your whiteboard a way of writing a calculation to show this pattern?

Support the child to record this array as: $3 + 3 + 3 + 3 = 12$

(and, if appropriate, $12 = 3 + 3 + 3 + 3$ and four columns of three showing four groups of three).

Repeat this activity using coloured counters to represent columns on previously prepared grids to encourage the child's fluency in:

- counting in steps of the column size;
- describing arrays as repeated addition;
- describing arrays as multiples of the column size.

? Tell me something you learnt today.

? Did you learn any new words? What were they?

Spotlight 1

Does not focus on 'rows of' or 'columns of', but only sees an array as a collection of ones

Opportunity for: solving problems

Toppling towers

Time 10–20 minutes

Resources

- Interlocking cubes (approximately 2 cm edge length)

Key vocabulary

rows	multiplied by
columns	pattern
array	groups of
arranged	towers
count how many	

Teaching activity

Explain to the child that this activity is going to help them to learn about using rows and columns to help them count more easily.

Make five towers, each of four cubes, each tower a different colour. Stand the towers upright as a vertical 4×5 array, and then lay the first two towers flat to form an array 4×2 on the table in front of the child.

? How many cubes are there in each of these towers?

? How many cubes are there altogether in the two towers that have fallen over?

With the child, rehearse 'four and another four so that makes eight altogether'. Get the child to repeat the description you have just given and emphasise the usefulness of this way of looking at the towers to find a quicker way of counting the total number of cubes.

If the child finds counting in fours too challenging, change the size of the towers to work on the same activity with towers two cubes high, making arrays 2×2 , 2×3 , etc. Count in twos to find the total number of cubes in 2×2 , 2×3 , etc.

Explain that you are going to try out this way of counting more efficiently using more of the towers. Add another tower to the array already laid on the table to make an array 4×3 .

? What is a quick way for you to find how many cubes there are altogether in the three towers?

Support the child in counting four and another four makes eight and another four makes twelve.

Repeat the activity, increasing the size of the array by adding one more tower of four cubes each time.

? Tell me something you know about rows and columns.

Spotlight 2

Does not focus on ‘rows of’ or ‘columns of’, but only sees an array as a collection of ones

Opportunity for: developing mental images

Egg boxes

Time 10–20 minutes

Resources

- Empty egg boxes with lids (that have contained four, six, ten, twelve eggs)
- Marbles (as large as possible)

Key vocabulary

rows	arranged
columns	count how many
array	multiplied by
pattern	groups of

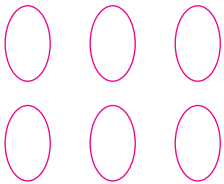
Teaching activity

Explain to the child that this activity is going to help them to learn about using rows and columns to help them count more easily.

Give the child one of the empty egg boxes which would have held six eggs, keeping the lid closed.

? How many eggs would this box hold, do you think?

Still with the lid closed, ask the child to show how the eggs would be arranged inside the box.



? How many in each column? (2)

? How many in each row? (3)

If the child cannot imagine the layout of the eggs inside the box, turn over the box and show them the humps in the base of the box as a hint. Repeat the questions about numbers of rows and columns.

Open the lid of the box and give the child the number of marbles which represent the number of eggs that they expected the box to contain. Guide the child to place the marbles in the indentations in the box, column by column, checking their original prediction. Emphasise the column structure. Repeat with their row prediction, emphasising the row structure.

If the child’s expectation of total number (or column or row size) was incorrect, encourage them to correct themselves, using the marbles to support their thinking.

As time allows, repeat the activity with different sizes of egg box.

? What shall we write about rows and columns that you think is important?

Spotlight 3

Does not focus on 'rows of' or 'columns of', but only sees an array as a collection of ones

Opportunity for: relating pictures to symbols

Footprints

Time 10–20 minutes

Resources

- Mixture of objects which include arrays, e.g. plastic minibeasts, such as insects, centipedes and spiders, and/or building bricks
- Play dough or small sand tray or paint-on sponge
- Counters
- Mini whiteboard and pen

Key vocabulary

rows	arranged
columns	count how many
array	multiplied by
pattern	groups of

Teaching activity

Explain to the child that this activity is going to help them to learn about using rows and columns to help them count more easily.

Choose an item such as a plastic ant (or other insect) and ask the child to look carefully at the arrangement of the legs and 'feet', when the ant is viewed from underneath.

? How many pairs of feet has it got?

? How many feet on each side?

? How many feet has this ant (insect) got altogether?

Make an imprint of the creature's feet in play dough, sand, paint or any other convenient medium.

Ask the child to recreate the arrangement of footprints that they see as an array made from counters.



? Tell me about the counter pattern you can see.

Encourage the child to observe the column and row sizes and match these to pairs of feet and numbers of feet on either side of the body respectively. With the child, count in twos to the total number of feet, highlighting two and two more and two more makes six feet altogether.

? Show me a calculation to add up the pairs of feet.

Support the child to record $2 + 2 + 2$ and 3 pairs of feet/counters. Demonstrate the match between the numbers in the calculation and the counter pairs, imprint pairs and feet pairs.

If the child has not inserted an equals sign in their recording, ask:

? How can you complete the number sentences to show the total number of legs on the insect?

Support the child to complete the calculations they have started as: $2 + 2 + 2 = 6$

3 sets of $2 = 6$, etc.

With the child, count in threes the total number of feet.

? Three on one side and three on the other makes how many altogether?

? Show me a calculation to add up the legs on either side of the body.

Support the child to record $3 + 3$ and 2 sets of feet/counters, with a record of the total count. Demonstrate the match between the numbers in the calculation and the counter positions, imprint positions and feet positions.

As time allows, select other objects to repeat the process of:

- identifying the array;
- reflecting on its structure;
- copying the structure in counters;
- counting to the total number in step size equal to the column size;
- describing the structure as numbers in rows and columns with associated repeated addition and multiplication calculations.

? Can you draw an array that is three rows of five? What size are the columns?

? What would you like more help with?

Spotlight 4

Does not focus on 'rows of' or 'columns of', but only sees an array as a collection of ones

Opportunity for: reasoning about numbers

.....

Jigsaws

Time 15–20 minutes

Resources

- 2 cm squared paper
- Paper grids made from 2 cm squared paper for the following arrays: 3×4 , 2×6 , 2×8 , 1×16 , 4×4 , 8×2 (for this activity, each will need to be cut into two parts)

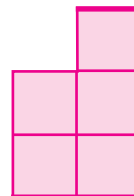
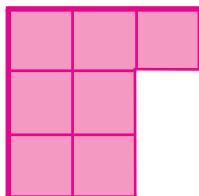
Key vocabulary

rows	arranged
columns	count how many
array	multiplied by
pattern	groups of

Teaching activity

Explain to the child that this activity is going to help them to learn about using rows and columns to help them count more easily.

Use a 3×4 grid cut into two parts, with heavy lines along the outside edges of the grid, for example:



Give the child one of the two parts (exactly as illustrated, without rotating it), explaining that you have another part to the grid and that you want them to work out what size it could be. Explain that the heavy lines show the outside edge of the shape.

- ? **How many rows do you think the missing piece has?** (*three*)
- ? **How many squares in each row?** (*three or more, or two or more, depending on piece selected*)
- ? **How do you know?**

Lay the part you have shown the child on a piece of 2 cm squared paper aligning the grid with the lines on the paper. Ask them to sketch the answers they have given you. Hide their sketch, just allowing them to see the piece you presented originally.

- ? **How many columns do you think it has?**
- ? **How many squares in each column?**
- ? **How do you know?**

Ask the child to check their sketch and make sure it still fits with their expected grid.

- ? **Could this be a piece from any other rectangular grid?**

If the child thinks other grids are possible, ask about these, encouraging the child to describe their ideas in terms of numbers of squares in the columns and number of columns. Then reveal the missing grid piece.

If the child is not sure about the range of possible grids, reveal the other part of the grid you have been discussing and ask the child to check its match with their idea. If their choice was not the piece that you revealed but nevertheless perfectly feasible, ensure that this is acknowledged and understood.

Support the child to show the ‘family’ of grids to which the piece could belong, emphasising the increasing number of columns.

Either repeat this activity with a part of another grid or provide a collection of grid pieces from four grids, each cut into two parts. Ask the child to recreate the four grids, explaining how they know they are correct. Ensure a continuing focus on the number in each column and the number of columns. If it seems appropriate, this could be an activity that the child takes home to try out.

Spotlight 5: a learning check

Does not focus on ‘rows of’ or ‘columns of’, but only sees an array as a collection of ones

Opportunity for: recognising relationships between numbers

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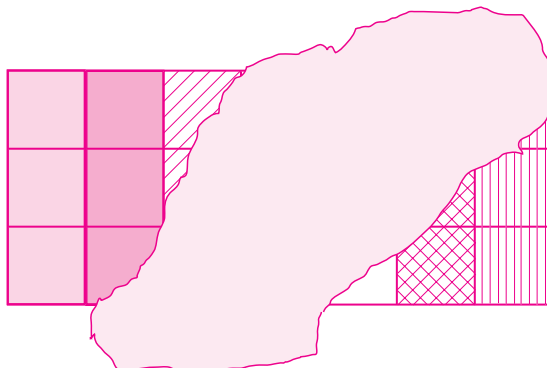
Chocolate bars Time 10–20 minutes

Resources	Check: does the child use key vocabulary?	
● Interlocking cubes (approximately 2 cm edge length)	rows	arranged
● Bar of chocolate	columns	count how many
● Scraps of paper to cover parts of rectangular arrays made from interlocking cubes	array	multiplied by
● Counters	pattern	groups of

Teaching activity

Explain to the child that this activity is going to help them to learn about using rows and columns to help them count more easily.

Make a rectangular array, for example 3×7 , from interlocking cubes, with each column a different colour from the adjacent columns. Cover part of the array with an irregular scrap of paper to hide parts of every row and two or three columns, for example:



? How many cubes are there in each column?

? How many cubes in each row?

Ask the child to show the pattern that they see, using counters to build up the array column by column.



Repeat the activity, making up other grids from assorted colours of cubes so the child does not have the cue of individually colour-coded columns. Use various sizes of paper scraps to cover parts of the arrays. Each time ensure that the child can recreate the array using counters.

Finish the activity by using a real chocolate bar, marked in squares!

Learning outcomes

By the end of this set of activities children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- recognise the rows and columns of an array;
- understand the organisation of objects into an array to support counting.

Has difficulty relating multiplying by two to known facts about doubles; records double four as $4 + 4$

Opportunity for: developing mathematical language

Resources

- Interlocking cubes
- Number cards (Resource sheet 1)
- *Doubling cards* (except 'is not') (Resource sheet 33)

Key vocabulary

double	match
multiply by two	double the length
tower	equals

Teaching activity

Time 15–20 minutes

Explain to the child that this activity is going to help them to learn that when they double a number or quantity, it is the same as multiplying by two.

Show the child a tower built of four cubes, all the same colour. Lay the tower on the table. Ask them to find the matching digit card.

Lay a tower of eight cubes, made of two colours, next to the first one. Ask them to find the matching digit card.



Pick up the first tower and place it alongside the second to illustrate that there are two lots of four.

? How many short towers do you need to make the long tower?

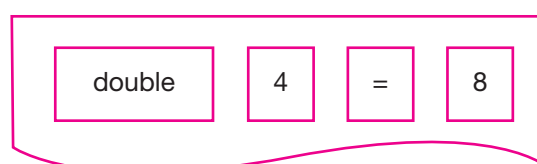
Check whether the child articulates this as 'two' and not 'one more'.

If the child has not identified the multiplying factor as two, but is focused on adding on, repeat the activity, drawing around the short tower twice to model the equivalence of two short towers to make the longer one. Remove the cubes, leaving the drawing and show that the longer tower matches the drawing.



Repeat using short towers of two and three cubes.

Place the 4 and 8 number cards with space between and around them so that the child can choose appropriate vocabulary cards to describe the operation. Lay all the vocabulary cards face up on the table. Ask the child to choose cards to make correct number sentences. Record these as they are created, for example:



? Where do you put the '× 2' (i.e. 'multiplied by 2') and '=' cards to make a number sentence?

4	× 2	=	8
---	-----	---	---

If the child is unable to locate the correct positions for the vocabulary cards, ask them to read out the number sentence they think they have created, giving them the opportunity to correct themselves. If they cannot correct themselves, return to the models and lay them on the table to make a picture number sentence.

					×2		=							
--	--	--	--	--	----	--	---	--	--	--	--	--	--	--

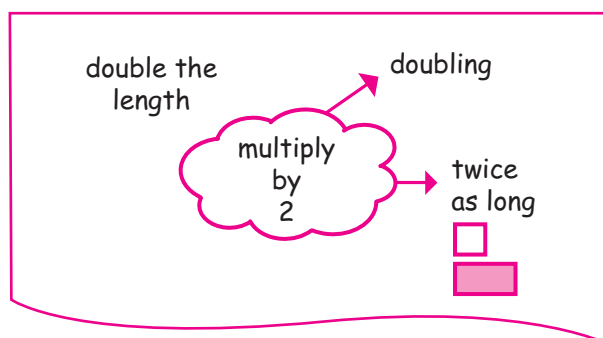
Ask the child to draw a new card from the pack of number cards and make a tower that matches it, and one that is double the length. They also need to choose the number cards to match the two lengths made.

? With your new numbers and your vocabulary cards, make all the number sentences you can for your new towers.

Support the child in recording each number sentence they make.

Repeat the activity with other numbers.

? Have you learnt anything about doubling today that we could write down?



Spotlight 1

Has difficulty relating multiplying by two to known facts about doubles; records double four as $4 + 4$

Opportunity for: estimating and making connections

4a Y2 $\times / \div$

Money bags

Time 15–20 minutes

Resources

- Two money bags/purses
- Selection of real coins, at least two of each denomination
- 0–40 number line

Key vocabulary

- | | |
|-----------------|--------|
| double | twice |
| multiply by two | equals |

Teaching activity

Explain to the child that this activity is going to help them to learn that when they double a number or quantity, it is the same as multiplying by two.

Ask the child to close their eyes and choose four coins from the collection. With eyes open they should then find out the total value of the selected coins. Ask them to put their chosen coins in a money bag/purse.

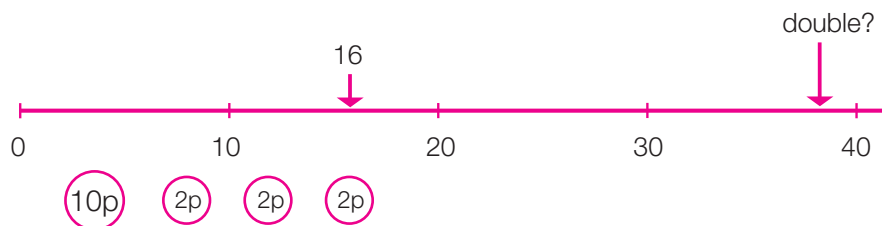
If the child cannot total the coins, offer a number line to support counting up the coins.



Ask the child to identify the total represented on the number line.



? Mark on the line where you think double the total would be. This is your estimate.



Show on the number line how the original was added to the same amount to make double the amount.

Ask the child to take the coins out of the bag and lay them on the table.

? How can you work out how close your estimate is?

Support the child in matching each of the coins on the table with a same value coin from the original collection.

? What is double the amount of the coins you put in the money bag?

? Where is the double amount on the number line?

Ask the child to put double the amount of money in the money bag/purse.

? How many times the original amount of money is in the money bag/purse now?

Help the child to articulate double the amount in the money bag as:

'Twice [*the original sum of money*].'

'[*The original sum of money*] multiplied by two.'

'We have two lots of each of the coins. We multiplied the coins by two.'

? What did you learn about doubling today? What could we add to your concept map?

Spotlight 2

Has difficulty relating multiplying by two to known facts about doubles; records double four as $4 + 4$

Opportunity for: reasoning about measures

.....

On the line

Time 10–20 minutes

Resources

- Two 0–30 number lines
- Number cards 0–9 (Resource sheet 1)

Key vocabulary

double	match
multiply by two	double the length
tower	equals

Teaching activity

Explain to the child that this activity is going to help them to learn that when they double a number or quantity, it is the same as multiplying by two. Ask the child to select a number card and mark the number which they have chosen on one of the number lines.

Indicate the distance along the number line defined by the chosen number card, using the distance between thumb and first finger.

? What is double the distance along the line?

? What is twice the distance along the line?

? Is there another way of asking this question?

Help the child to show double by using the distance between thumb and finger twice. Make marks on a number line to show the original distance and double the distance. Make the comparison between the two.

If the child is not clear about the connections between doubling, finding twice as much multiplying by two, take one of the number lines. Choose a digit. Fold the line at the marked point defined by the chosen digit and mark the position of double the original length:



continued

Place a corresponding number line underneath and identify the point at which the edge of the original line finishes.

? What is double the distance along the line?

? What is twice the distance along the line?

? What is another way of asking this question?

Repeat the activity by getting the child to use different starting digit cards and show where:

- twice the number;
- double the number;
- the number multiplied by two;

will be on the number line, by folding and marking the position of the doubled number.

? Is there anything we could add to your concept map about doubling and multiplying by two?

Spotlight 3

Has difficulty relating multiplying by two to known facts about doubles; records double four as $4 + 4$

Opportunity for: exploring mathematical language

Double drinks

Time 10–20 minutes

Resources

- Jug of coloured water
- Measuring cylinder or straight-sided clear containers
- Empty cups (of the same type and size)
- Marker pen to write on glass
- *Doubling cards* (Resource sheet 33)

Key vocabulary

double	two lots of
multiply by two	not
double the quantity	twice

Teaching activity

Explain to the child that this activity is going to help them to learn that when they double a number or quantity, it is the same as multiplying by two.

Fill two cups with coloured water and pour each one individually into a measuring cylinder.

? Where do you think the liquid will reach if you pour in double two cups?

Support the child in using their fingers to estimate the new height of the coloured water, and mark it on the side of the cylinder with a pen.

? How did you choose where to put the mark?

Ask the child to pour in some more coloured water to double the amount in the measuring cylinder.

Place the vocabulary cards face down on the table and ask the child to turn over a card.

? Tell me what we have just done. Try to make sure you use the word(s) on the card you've chosen.

Encourage the child to turn over another card and explain using the new word as part of their explanation.

Repeat the activity using different numbers of cups of coloured water each time.

? Did you learn anything new today that we can add to your concept map?

Spotlight 4

Has difficulty relating multiplying by two to known facts about doubles; records double four as $4 + 4$

Opportunity for: solving real-life problems

Plenty of time

Time 10–20 minutes

Resources

- Teaching clock (analogue)
- *Activity cards* (Resource sheet 34)
- Mini whiteboard and pen

Key vocabulary

double	match
multiply by two	double the length
tower	equals

Teaching activity

Explain to the child that this activity is going to help them to learn that when they double a number or quantity, it is the same as multiplying by two.

Ask the child to choose a card from the activity card pack. According to the activity selected, decide upon a suitable start time (on the hour, half-hour or quarter-hour) and a suitable length of time for the activity (in multiples of a quarter-hour).

Write the start time and the length of the activity on the whiteboard for reference alongside the selected activity card, for example:



Support the child in showing the start time on the clock. Ask the child to watch as you move the hand to the finish time.

Explain that Tim always takes a long time: always double the time his brothers and sisters take (we could say 'he takes twice as long'). Ask the child to show how long Tim would take by turning the hands of the clock.

? What time will Tim finish [insert relevant activity]?

If the child finds this too challenging, use examples that start and finish on the hour.

Repeat using other activity cards, or get the child to choose activities (and start times and durations, if appropriate).

? Can you give me a sentence that includes 'twice as long'?

Spotlight 5

Has difficulty relating multiplying by two to known facts about doubles; records double four as $4 + 4$

Opportunity for: explaining and discussing

Double it

Time 15–20 minutes

Resources

- Card strips of different lengths, for example 3 cm, 6 cm, 2 cm, 4 cm, 8 cm, 5 cm, 10 cm (two or three of each)
- *Doubling cards* (Resource sheet 33)
- Number cards (Resource sheets 1 and 2)
- *Dotty cards* (Resource sheet 9)
- Cubes
- At least one other child

Check: does the child use key vocabulary?

double	double the length
multiply by two	equals
tower	is not twice
match	

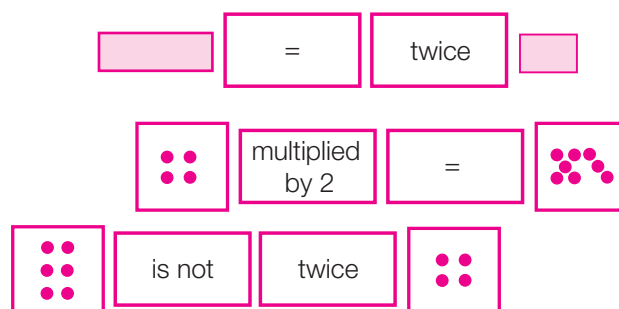
Teaching activity

‘We are going to play a game, **Double it**, today. It is about doubling being the same as multiplying by two.’

Put the card strips, cubes, dotty cards and number cards face up on one side of the table, and the vocabulary cards on the other side. The game is intended for cooperating pairs, but children could race each other.

How to play

1. Players take a vocabulary card which they think they can make a sentence with, for example, ‘multiplied by 2’.
2. They then select a card strip, dotty card, cube ‘train’, or number card to make a ‘picture’ number sentence, for example:



Continue the activity, checking the players’ understanding by asking them to explain their picture sentences.

Variations

- Use only number cards and try some larger numbers.
- Race to make three sentences in 5 minutes.

Learning outcomes

By the end of this set of activities children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- make the link between doubling and multiplying by two;
- record number sentences using double or multiply by two interchangeably.

Does not use partitioning to find double twelve or double thirty-five

Opportunity for: developing mental images

Resources

- Number cards 0–9 (Resource sheet 1)
- *Doubling cards* (Resource sheet 33)
- Counters/interlocking cubes
- Additional + cards (Resource sheet 8)
- Base 10 apparatus
- Place value cards (arrow) cards

Key vocabulary

- | | |
|--------|-----------------|
| double | multiply by two |
| equals | partitioning |

Teaching activity

Time 15–20 minutes

Explain to the child that this activity is going to help them to learn more about doubling and how to use partitioning to help with doubling.

Ask the child to double small numbers below five. Record the doubles that they recall, for use later.

Double 1 = 2
Double 2 = 4

? Can you double eight?

? How did you do it?

If the child uses partitioning strategy, repeat for other numbers below ten, then move on.

If the child does not use partitioning, but counts on from eight, explain that you are going to work on learning other ways of doubling numbers so that the child does not always need to count on to work out a double.

Provide counters or interlocking cubes and ask the child to show you all the different ways in which they can make eight. Ask the child to find all the possible ways of adding two numbers to make eight and lay them out on the table:



Then ask the child to match digit cards to each set: $8 + 0$, $7 + 1$, $6 + 2$, $5 + 3$, $4 + 4$.

Explain that you are going to work together, using these partitioning facts to double eight.

Show double eight with eight counters partnered by another eight:



Use this layout of counters to illustrate how double eight is equal to, say, double five plus double three.

? You know that four and four equals eight. How could you use this number fact to help you find double eight?

Ask the child to illustrate their calculation as double four and double four again, using the counters to show their thinking.

Repeat this activity using other partitions of eight to find double eight.

? Which was the easiest number fact for you to use in partitioning to find double eight?

? Why was this easiest?

Use the child's answers to these questions to reinforce that they can choose any way of partitioning the number they have to double, using number facts that they know.

Repeat the activity with numbers between five and ten, other than eight.

If the child needs reinforcement of doubling numbers below ten using partitioning, repeat the activity above for a range of numbers between three and ten. Ensure that the child has the opportunity to comment on the fact that the different partitionings of an individual number all yield the same double.

Spotlights 1 and 2 provide further contexts for doubling numbers up to ten using partitioning.

? What is double ten? What is double two?

? Can you tell me what double twelve is?

If the child is able to make use of this partitioning to double twelve, ask them to choose another number between ten and twenty and use counters/cubes, or a diagram which they create, to show how they partition to calculate double their chosen number.

If the child has difficulty working out double twelve, use interlocking cubes or Base 10 apparatus to partition teen numbers into ten and ones. Use place value cards to label each group. For each number, ask the child to make the pattern twice, so that they have a visual picture of the doubles. Match cards to these groups as well.

Combine the tens groups.

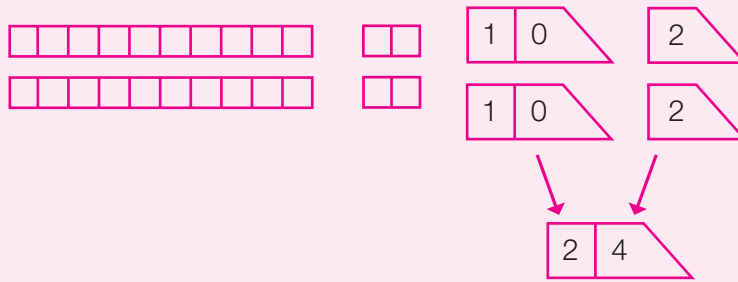
? Which arrow card can we replace the two tens with?

Combine the ones groups.

? Which arrow card can we replace the two twos with?

Then combine the groups of apparatus to illustrate twenty-four and the arrow cards to label twenty-four.

continued



? What number have we doubled?

? How can you find double twenty-three?

Offer the child Base 10 apparatus and place value cards to help them illustrate the process they use to find double twenty-three.

If the child cannot extend the process to numbers above twenty, help them to make a link to their success with numbers up to twenty and then choose a range of numbers between twenty and thirty to partition into the 'tens' and 'ones' parts, using place value cards to illustrate the process.

Ask the child to choose numbers to partition and double.

? We did something very special today to find doubles. Can you tell me what we did?

Spotlight 1

Does not use partitioning to find double twelve or double thirty-five

Opportunity for: reasoning about numbers

.....

Handy fingers

Time 10–15 minutes

Resources

- Dice numbered 5–10
- Number cards 0–10 (Resource sheet 1)
- Mirror

Key vocabulary

double	twice
multiply by two	equals
plus	

Teaching activity

Explain to the child that this activity is going to help them to learn about using their fingers to help them double.

Ask the child to throw the dice and show the number on their fingers, with five on one hand and the rest of the number on the fingers of the other hand, for example, five on one hand and three on the other.

? What number sentence are you showing me?

Repeat until the child is confidently showing and saying a ‘five plus something’ sentence each time, for example, ‘eight is five plus three’.

Ask the child to choose a number card and show the number on their fingers with five on one hand.

? Now can you double the number?

Ask the child to hold their hands up to the mirror so that the number is represented twice.

Point out that the five can be seen twice, as can the other number, say three.

? How many fives can you see? What is double five?

? How many threes can you see? What is double three?

? So what is double eight?

Repeat this activity for all the numbers in the pack, removing the mirror when the child thinks they can manage without it.

Continue until the child can double using only the model of their own fingers and imagining the double.

? How does using your fingers like this help you to double?

? Do you think you could practise this at home so that you learn some doubles by heart?

Spotlight 2

Does not use partitioning to find double twelve or double thirty-five

Opportunity for: making decisions

.....

Double time

Time 10–20 minutes

Resources

- Number cards 0–10 (Resource sheet 1)
- Sand timer

Key vocabulary

double	add
multiply by two	plus
partitioning	equals

Teaching activity

Explain to the child that this activity is going to help them to learn about doubling numbers up to ten.

Give the child the 5 card. Spread the 1, 2, 3 and 4 cards face down for the child to choose another card at random to add to the 5 card.

? What is the total of your two cards?

Give the child the number card that represents their total and lay it on the table with the other two cards, for example:



? How can you work out double the total?

Support the child in doubling the five and doubling the other number in order to find double the total.

Repeat with other cards drawn randomly to accompany the 5 card.

If the child can recall quickly how to work out doubles, challenge them to beat their own time against a sand timer, doubling as many numbers as they can before the sand runs through.

? What do you think you are really good at now?

? What would you like more help with?

Spotlight 3

Does not use partitioning to find double twelve or double thirty-five

Opportunity for: reasoning about numbers

.....

Double time again

Time 10–15 minutes

Resources

- Number cards 0–20 (Resource sheets 1 and 2)
- Sand timer

Key vocabulary

double	tower
multiply by two	match
double the length	equals
partitioning	

Teaching activity

Explain to the child that this activity is going to help them to learn about doubling numbers up to twenty.

Give the child the 10 card. Spread the 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 cards face down for the child to choose another card at random to add to the 10 card.

? What is the total of your two cards?

Give the child the card that represents their total and lay it on the table with the other two cards, for example:

10	9	19
----	---	----

? How can I work out double the total?

Support the child in doubling the ten and doubling the other number in order to find double the total.

Repeat with other cards drawn randomly to accompany the 10 card.

If the child can recall quickly how to work out doubles, challenge them to beat their own time against a sand timer, doubling as many numbers as they can before the sand runs through.

? What do you think you are doing well?

? What could you do better?

Spotlight 4

Does not use partitioning to find double twelve or double thirty-five

Opportunity for: applying mathematics in a real-life context

.....

Double your money

Time 10–20 minutes

Resources

- Either objects or picture cards, with items priced from 10p to 50p
- A collection of coins – all denominations up to £1 coin, with a number of coins of each type, except for 50p and £1 coins

Key vocabulary?

double	equals
multiply by two	partitioning
how much does it cost?	

Teaching activity

Explain to the child that this activity is going to help them to double numbers up to fifty.

Ask the child to select an item.

? How much does it cost?

? How much would you have to pay for two?

Encourage the child to select the coins necessary to pay for the item. They can then check their answer to the second question by doubling the coins and counting.

Repeat using a different priced item each time.

If the child has difficulty with any particular numbers, ask them to explain their method.

Are they using 10p coins as part of their partitioning strategy?

Check that they are not counting on in ones, but are also using the five plus a bit strategy by choosing 5p coins.

? Which numbers are harder for you than others? Why do you think that is?

? What do you think is your best method for finding doubles?

Spotlight 5

Does not use partitioning to find double twelve or double thirty-five

Opportunity for: explaining and discussing

.....

Doubles bingo

Time 15–20 minutes

Resources

- At least two children
- Mini whiteboards for each child
- Two sets of number cards 0–50 (Resource sheets 1, 2, 3 and 4)
- Bag

Check: does the child use key vocabulary?

- | | |
|-----------------|--------------|
| double | equals |
| multiply by two | partitioning |

Teaching activity

‘Today we are going to play a game called **Doubles bingo**, and this game will help you to get even better at doubling numbers up to fifty.’

Divide up the number cards so that you have set 1 (all the even numbers from 10–50) face up on the table.

The set 2 cards are all the odd numbers from 5–25. You will use these to call the numbers, so they should be face down in a pile, or in a bag.

The players can cooperate in pairs or play against each other.

How to play

1. Each player chooses four cards from set 1, and writes these four numbers on their mini whiteboards.
2. Explain that you will call out a number from set 2 and that the players have to double that number and see if the answer is one of the numbers which they have written on their whiteboards. If it is, they cross it out. Put the card used to one side.
3. The first player to cross out all their numbers is the winner.
4. Ask the players to check that the numbers have been crossed out correctly.

Variations

- A child can be caller.
- Extend the numbers to 100.



Play a whole-class game with even numbers from one hundred and thirty to one hundred and fifty selected by the children and written on their whiteboards. Call out odd numbers from sixty-five to seventy-five.

Learning outcomes

By the end of this set of activities, the children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- double numbers by partitioning into parts for which doubles are known.

Does not use knowledge of doubles to find half of a number; for example, continues to find half by sharing using a 'one for you' approach and cannot apply knowledge of doubles

Opportunity for: applying previous knowledge

Resources

- Cubes
- Bead strings
- Number cards to 20 or 30 (Resource sheets 1, 2 and 3)
- Number line to 20 or 30

Key vocabulary

half	doubling
halving	opposite
double	undoes

Teaching activity

Time 10–15 minutes

Explain to the child that today's activity will help them to be better at finding half of a number, and that at first they will be using some of the things which they already know about doubling.

? What do you think doubling a number means? Can you use these cubes to show me?

Follow on from what the child says. Try to establish how the child doubles a number such as nine.

Do they use partitioning, splitting nine into five and four then doubling those numbers?

(See 4a and 4b Y2 $\times/\div$.)

If the child is struggling with this, ask them to make cube 'trains' of teen numbers with a ten part of one colour and the units a different colour (so thirteen might be ten red and three blue cubes).

Remind them of partitioning numbers into five and a bit to find doubles. (See 4a and 4b Y2 $\times/\div$.)

Demonstrate how to halve cube 'trains' into five and a bit.



'Half of fourteen is five and two more. If you double five and add double two you get back to fourteen.' Move the cubes to fit in with your words, showing halving undoing doubling.

Explore doubles and halves for a range of different numbers, using cubes or a bead string, and jot down some numbers which the child can double.

If the child is struggling with the doubling, use just a few low numbers for doubling, for example four, six, eight and ten.

Using fingers to show five and a bit numbers might help. So six is five fingers on one hand and one on the other. If you use your fingers too, you can show double six as $5 + 1$ added to $5 + 1$.

That will make one lot of ten and two more.

Then try doubling seven in the same way. Five fingers on one hand and two on the other. So double seven is $5 + 2$ added to $5 + 2$.

Record some of the doubles on paper in a range of sentences, using the key vocabulary above.

Five doubled is ten
Double five is ten
Ten is double five

Then move on to halving, at first using the numbers which the child seems most confident with.

? Can you tell me what half of six is? How will you work that out? How do you know you are right?

If the child can halve six, move on to some other even numbers, offering the child cubes.

Observe what the child does, then try a larger number.

If the child needs support (and followed the use of fingers to make five and a bit), return to that and record some of the halving they can do. Then make links to the larger numbers.

? How would you find half of twelve? ...eighteen? ...twenty-four? ...twenty-eight?

If the child uses the 'one for you' method of sharing, try to make links with other halves which the child knows. For example, if they know that half of ten is five, suggest to them that half of twelve must be very close to five, because twelve is so close to ten on the number line. Make the link from half of ten is five to half of twenty is ten, so half of eighteen must be close to ten.

Help children to make the links between what they said to you at the start of the lesson about doubling and the ideas that they seem to be struggling with now.

Look at the doubling written above with the child and then write a related sentence.

Five doubled is ten

Half of ten is five

Write some other halving sentences, always relating them to the doubling sentence. For example: 'Twenty halved is ten.', 'If you halve twenty you get ten.', 'If you double ten you get back to twenty.'

If the child seems not to understand, even with the lower numbers, you can demonstrate this with bead strings. 'Three doubled is six. Six halved is three.'



Do as many additional numbers with cube 'trains' or beads as you have time for.

You can record what the child does to remind them another time, or ask the child to record it in their own way. (This can often give you an insight into the child's thinking and any errors which they might still be making.)

? What would you like me to record about our lesson today?



? How would you use partitioning to help you to find half of a number? Would partitioning help you to find half of a number like thirty-seven?

Spotlight 1

Does not use knowledge of doubles to find half of a number; for example, continues to find half by sharing using a 'one for you' approach and cannot apply knowledge of doubles

Opportunity for: seeing connections between numbers

Round and rounds

Time 10 minutes

Resources

- Cubes
- Bead strings
- Prepared 'round and rounds'
- Two sets of number cards 0–55 (Resource sheets 1, 2, 3 and 4)
- Top hat and toy rabbit (optional)

Key vocabulary

half	doubling
halving	opposite
double	undoes

Teaching activity

Explain that today we are going to explore some numbers and find something surprising about doubling and halving.

? Can you tell me what you already know about doubling and halving?

Have cubes or bead strings available and show the number seven.



Seven is half of fourteen.



? Can you show me how you can use the beads to show double seven? What happens if you halve the answer?

? Do you think that will happen with other numbers?

Try with some other numbers.

If the child is not seeing the relationship between the doubling and halving, use some smaller numbers, such as finding half of four or six.

If the child still seems a bit unsure, move on anyway, but keep to lower numbers and you might need to go back to *4a* and *4b* Y2 $\times/\div$ to do some more 'five and a bit' halving and doubling.

Write one of the number sentences that the cubes or bead strings illustrate.

Ten doubled is twenty;
then if you halve twenty,
you get ten.

Make quite a bit of this! You could call it 'number magic' if you want to! Write another sentence but with spaces for number cards.

doubled is;

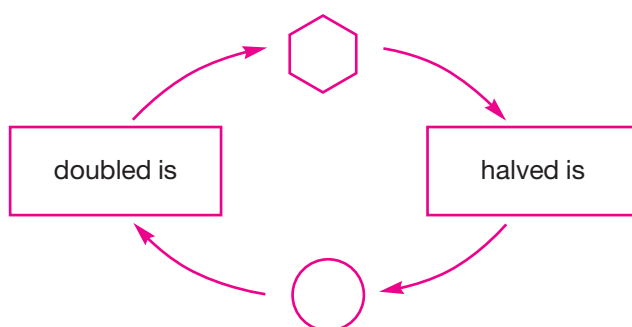
then if you halve, you get.

Spread the number cards out on the table.

? Can you find four number cards to fit in the spaces?

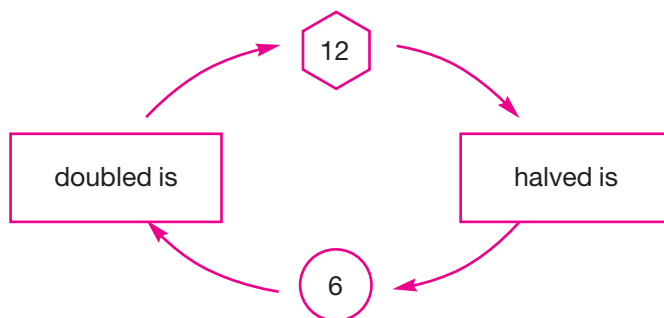
Establish that, although you needed four cards, you only used two numbers.

Introduce 'round and rounds', showing one 'round and round' with no numbers. Ideally pull a prepared one out of a top hat! (Keep the rabbit hidden for now.)



? Can you move just two of your cards from the number sentence to the 'round and round' and do some number magic? Now read the sentence to me, starting at the top in the hexagon.

Stress that you can keep on going round and round. Halving is the opposite of doubling. Doubling is the opposite of halving. Halving undoes doubling. Doubling undoes halving.



If the child doesn't seem to be coping with this, try more halving and doubling in odd moments of class time. (The rabbit and top hat might help. Each day you could put two number cards, such as 5 and 10, in the hat with the rabbit and ask the child to make a 'round and round', talking you through what they have done.)

? What did you learn today about halving and doubling numbers?



? Will 'round and rounds' work for any number? Why do 'round and rounds' work?
(Try to get at the idea that doubling and halving 'undo' each other.)

- ? Can you go around the other way on 'round and rounds'? (Yes.)
- ? What would you need to change to make this work?
- ? What 'number magic' could you do with one and a half and three?
- ? Which other numbers could you use with one and three quarters to do some 'number magic'?

Spotlight 2

Does not use knowledge of doubles to find half of a number; for example, continues to find half by sharing using a 'one for you' approach and cannot apply knowledge of doubles

Opportunity for: recognising patterns and relationships and developing mental images

Half back jumps

Time 10–20 minutes

Resources

- Dominoes
- Cubes
- Number cards 0–10 (Resource sheet 1)
- Wipe-clean number line
- 'Washing line' and pegs (optional)
- Small character toy or puppet

Key vocabulary

half	doubling
halving	opposite
double	undoes

Teaching activity

'We're going to do some half back jumps with this puppet to show jumping back halfway.'

Ask the child to pick out the double dominoes.

? How did you know that was a double?

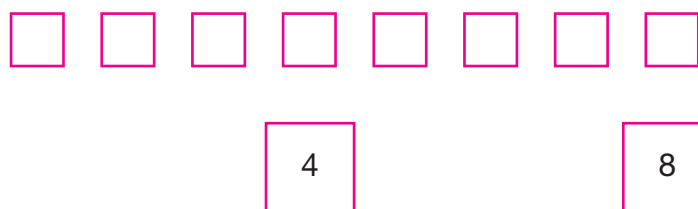
? If each part of the domino has four spots, how many spots for double four?

? How many spots for half of eight?

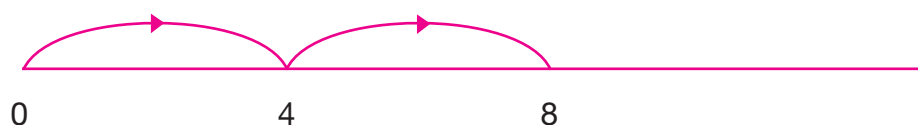
If the child is struggling with this, repeat with several different numbers. You could remind them of the 'round and rounds' they did before (see *Spotlight 1*).

Ask the child to put out cubes in the same pattern as a double four domino. 'Now move them into one long line.'

Lay number cards at the halfway point and at the end of the line.



Now link this to jumps along the number line with the puppet.



Ask the child to get the puppet to jump back halfway.

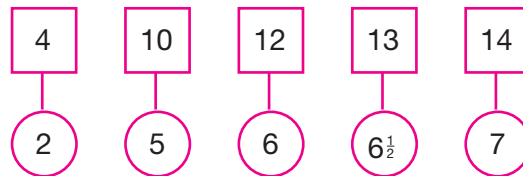
? What was the right number for your puppet to jump back to?

? So what is special about four and eight?

If the child cannot explain that four is half of eight, do some more examples with low numbers, using the dominoes if that seems to help the child.

You could record these doubles and halves along a washing line, pegging the 'half' number underneath.

Continue the activity of jumping back along the number line, using numbers which the child can cope with most of the time, but always challenge (at least for some of the time).



? If the puppet stands on 14, where will it land if it takes a halfway back jump?



Ask the child to record in any way they want what they learned today.

Do some half back jumps for some larger numbers, as appropriate, for example 1000, 500 000, 1 000 000, 4254.



? Are some numbers easier to halve than others? Why? What is special about halving odd numbers?

Spotlight 3

Does not use knowledge of doubles to find half of a number; for example, continues to find half by sharing using a 'one for you' approach and cannot apply knowledge of doubles

Opportunity for: exploring real-life contexts

.....

Half each

Time 15–20 minutes

Resources

- Lengths of 2 cm squared paper (up to 40cm long)
- Coins (of various denominations)
- Teaching clock
- Metre ruler

Key vocabulary

half	doubling
halving	opposite
double	undoes

Teaching activity

Explain to the child that today they are going to do some doubling and halving, because that is an important thing to be able to do. Ask them to imagine that they have to share some money between themselves and a friend, and they want to share it equally.

Give the child a small amount of money that you think they could halve, for example, six pence.

? Can you find half of this? What will you do?

**? How do you know that three pence is half of six pence?
What if you double three pence? What happens?**

? Which other amounts of money could you halve?

If the child is happy halving amounts with just 1p coins, introduce some other coins and find half. Always make the link back to the amount of money which they started with being double the answer that they found when they halved, recording at least one of the calculations with the child.

? Which amounts of money do you find easy to halve?

? What is the biggest amount of money that you think you could halve?

Try to get to half of a pound and half of fifty pence.

Using a strip of 2 cm squared paper, for example, eighteen squares long (36 cm), ask the child to fold it in half.

? How many squares long is half? Now open it up so you get double the length.

? How many squares long is it now?

Repeat with a strip twenty squares long (40 cm).

If the child is struggling with the numbers, use shorter lengths of paper.

Now ask the child to shut their eyes and try to imagine a strip of paper twenty-two (or twenty-four or twenty-six) squares long.

? If you fold it in half, how many squares long will it be now?

If appropriate, using a teaching clock, show the hands at 12, then move them half way around.

Ask the child to put both hands at an o'clock time which they can read, for example, nine o'clock.

If the child is not familiar with reading an analogue clock, note that for a future time and move on.

Now ask the child to show half past nine. The long minute hand has gone half way around the circle.

You could count the minutes in fives if the child seems to know how to read some clock times. Start at 12 with zero, and count in fives to half past.

Confirm that this is thirty minutes past.

? How many minutes will there be to go around the other half of the clock?

? How many minutes are there altogether in one hour?

? Let's find 60 cm on this metre ruler. Can you find half of 60?

Finish by asking the child which numbers they can now find half of in their head.



Choose a 'difficult' amount of money, for example £17.39, and ask the children to find half of it.

Spotlight 4

Does not use knowledge of doubles to find half of a number; for example, continues to find half by sharing using a 'one for you' approach and cannot apply knowledge of doubles

Opportunity for: investigating numbers

.....

Numbers in my head

Time 10–15 minutes

Resources

- Cubes in 'ten trains' of all the same colour and units
- Large book or folder and two pots
- Number line

Key vocabulary

half	doubling
halving	opposite
double	undoes

Teaching activity

'Today we are going to do some games where you have to guess the number that I'm thinking of.'
Stand a big book on the desk propped open to hide what you are doing with the cubes.

? I've got four cubes in this pot and four in this one. How many cubes have I got altogether?

Repeat with other numbers. Then give the child ten cubes.

? You have double the number of cubes I have in this pot. How many cubes are in this pot?

? What is half of ten? What is double five?

Repeat with some other numbers and record some doubles and halves for the child.

If the child is still finding it hard to find halves, let them make double and half 'trains' of cubes and keep these on display for a few days. For example, a 'train' of ten red and four blues, and half that, five red and two blues.

Give the child some 'ten trains' and single cubes and move on to some numbers that challenge the child.

? The number in my head is half of sixteen. What is my number?

? I doubled a number in my head and the answer was fourteen. What was my number?

? My number when it is halved then halved again is two. What is my number?

Remind the child that there are sixty minutes in an hour. Let them count out six 'ten trains'.

? How many is half of sixty? What is double thirty?

? The number in my head is half of a hundred so what is my number?

? The number in my head is half of two million. What is my number?

Talk with the child about other numbers on the 100-square that might be easy to halve.

? Can you explain why some numbers are easier for you to double and halve than others?



'Make a list of three numbers between one hundred and two hundred that have three different digits and are easy to halve, for example, one hundred and twenty-four.'

‘Make a list of three different numbers also between one hundred and two hundred that are harder to halve.’

? Why are these numbers harder to halve?

Spotlight 5: a learning check

Does not use knowledge of doubles to find half of a number; for example, continues to find half by sharing using a ‘one for you’ approach and cannot apply knowledge of doubles

Opportunity for: discussing and explaining

Doubles and halves game

Time 10–20 minutes

Resources

Check: does the child use key vocabulary?

- At least one other child
- One or two sets of number cards 1–20 but take out 11, 13, 15, 17, 19 (Resource sheets 1 and 2)
- Paper and pen to keep score
- Cubes, number lines or bead strings

half	doubling
halving	opposite
double	undoes

Teaching activity

‘Today we are going to play **Doubles and halves**, so that you will become really good at remembering how to double and halve.’

Lay the number cards face up on the table. If you have space for two sets it will make the game a little longer; two sets will be needed for three or more players.

The aim of the game is to score the highest total.

How to play

‘If you choose carefully, you can get a really high score if you pick up pairs of cards that are doubles and halves of each other. So if you want to pick up 5, you can pick up 10 as well because it is double five.’

1. Players take it in turns to pick up two cards (or just one) of their choice that are doubles and halves of each other, for example 2 and 4.
2. They must explain why they have chosen those numbers – but if they say the wrong thing, they must put the cards back on the table!

Players can use number lines, or other means, to show why their cards make a ‘trick’.

If they say correctly why the cards make a trick (e.g. ‘five is half of ten’), these two cards then make a trick and the player puts the trick in front of them.

3. Other players then take a turn, putting each trick on the table until all the cards are used, or no more tricks can be made. (You might want to play so that when there are just a few cards left, everyone takes only one card at a time.)

4. Everyone helps everyone else to find their score. Scoring is as follows:
Two cards that are double and half of each other score 20.
A trick of three cards, for example 5, 10 and 20, scores 30.
A trick of four cards, for example 2, 4, 8 and 16, scores 40.
5. Keep the score carefully and play another round. Add each new score to make a running total and see if anyone can score over 200.

If a child seems not to be managing the game, play alongside them, or ask all the players to play with a partner. Use cubes, a number line or bead strings where support is needed.

? Why did you choose that number? Can you explain why those numbers make a trick?

Variations

- Pairs can cooperate to make the longest tricks they can. Use a set of cards for each pair.
- ↑ ● Extend the game to use other number cards, for example to 30. Put the 15 number card back in, but take out 21, 23, 27, 29.
- ↑ ● Sometimes play with all the numbers left in.

? When we leave all the numbers in, why do some numbers never get used for a trick?

? Can you continue with this halving sequence? 66, 33 ... You can use a calculator.

? What was the smallest number you got to? Will you get to a negative number eventually? (No.)

Learning outcomes

By the end of this set of activities, children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- use knowledge of doubles to find half of a number;
- use a range of strategies for halving in real-life situations;
- begin to halve and double confidently, and know some by heart.

Is not systematic when sharing into equal groups using a 'one for you' approach; does not use the language of division to describe the process

Opportunity for: developing mathematical language

Resources

- Biscuits, sweets or cubes
- Character toys or smiley faces
- Several sets of number cards (Resource sheets 1 and 2)
- Calculator

Key vocabulary

- | | |
|----------------|---------------------------|
| share between | equal groups |
| share into | same number in each group |
| divide between | fair |
| divide into | |

Teaching activity

Time 10–15 minutes

Explain to the child that today they are going to do some sharing with sweets and biscuits. Today's work will help them to be clearer about what division means. They will be learning some of the important words for division.

Note: In this set of activities just keep to 'divide' or 'share', 'between' or 'into' (sharing model for division). 'Divide by', using repeated subtraction (grouping model for division), is covered in the next set of activities, (7 Y2 $\times/\div$).

? Can you tell me what you think sharing into equal groups means? Can you show me using these sweets?

Build on what the child says, writing down some of the appropriate words which they use.

Set up a sharing story, such as three children sharing fifteen sweets.

? Can you give them all an equal share?



If necessary, discuss the meaning of 'equal'.

Observe the child closely to see how they do the sharing.

? Can you find the right number cards to put beside each person to show how many sweets they have each?

? Can you tell me what you've done? Let's make a number sentence about what you did.

If the sharing is not systematic, you will need to ask the child to count carefully how many sweets each person has.

? Is it fair? Do they have the same number of sweets each?

If the child still seems confused, gather all the sweets back into one pile and suggest that the child shares them out again, very carefully, to make sure that each person has the same number.

Encourage the child to talk you through what they are doing.

Record for them the numbers and words which they use, perhaps with a small picture of the three children.

Try some other numbers, such as eighteen sweets shared between three people, recording for the child using a wide range of division vocabulary.

It is important that they learn to describe the process of division, talking you through all their actions.

Eighteen sweets shared equally between three people makes three groups of six.

Eighteen sweets shared into three groups is six in each group.

If you share eighteen sweets equally between three people, they get six each.

Eighteen sweets divided between three people gives each person six.

If these numbers are too challenging for the child, you might want to stick to sharing between just two people and keep the numbers low until the child is more confident. It is important, though, to develop a wide range of vocabulary during the session.

Try with twenty-one sweets *between* three people and, if the child has become more systematic with sharing using 'one for you', suggest that they move on from giving out one at a time and give out two or more at a time.

? How many sweets do they have each? Are all the groups the same size? How do you know?

'Let's write down what you did. You tell me what to write.'

I started with twenty-one sweets and I divided them between three people. I gave them two at a time until I only had three left. When all the sweets were shared out, they had seven each.

? How many equal groups have you made?

? How many sweets in a group when you divide twenty-one sweets between three people?

? What is twenty-one divided between three?

(Avoid the phrase 'goes into'.)

Show the child a calculator and ask them if they know which key is the division key. If they know, ask them how they would key in twenty-one divided between three.

If the child doesn't recognise the division symbol, demonstrate on the calculator:

$$21 \div 3 = 7$$

Then relate this back to the practical world and their other sentences.

Keep all the sentences and a record of the numbers you have used with the child. These can be displayed, if you have space, or talk them through at another time in class. You will be building up a bank of number sentences and words throughout this teaching sequence.

If there is time, you could ask the child to draw one of the divisions they did.

? What did you learn today?

What were the really important words we used today?



? Write two division sentences in words: one that includes 'between' and another that includes 'into', using $34 \div 2 =$.

Spotlight 1

Is not systematic when sharing into equal groups using a 'one for you' approach; does not use the language of division to describe the process

Opportunity for: solving real-life problems

Dividing between two

Time 15–20 minutes

Resources

- Small character toys or smiley faces
- Items such as sweets, muffins, biscuits, pens
- Paper and coloured pens
- Number sentences from the previous activity
- Number cards 0–20 (Resource sheets 1 and 2)

Key vocabulary

share between	same number in each group
share into	fair
divide between	half
divide into	halving
equal groups	

Teaching activity

'Today we are going to do some sharing between two people using some of the objects on the table.'

Ask the child to show you what they can already do with sharing between two.

Listen to what the child says and note the words that they use, then build on this for the activity. (Again, keep a note of all the numbers which the child uses. These can be used to help the child to learn some divisions by heart.)

Put out two of the character toys and ask the child to count out some pens that they think will share equally between two with none left over. Let them do the sharing and observe their method for doing that.

If the child doesn't recognise that they need an even number of pens, put out the even numbers cards and explain that an even number is needed for two to share (10, 12, 14, 16, 18, 20, and so on). (Make a note to do some work on even numbers at a later date.) Explain that they can put any left-over pens back in the box.

Show how ten divided between two makes five each, but that eleven between two means one pen is left over.

If the child cannot work systematically with the sharing, show them how they could do it, talking through what you are doing. You can extend 'one for you' to 'two (or more) for you' each time.

Record the numbers for the child and talk through what they did.

I started with eighteen pens and
I shared them between two people.
They have nine each so it is fair.
There are two equal groups of nine.

Eighteen divided
between two people
makes two equal
groups of nine.

? Can you think of another way we can talk about sharing between two people?

If the child cannot make the link to halving, ask them if they think each person has half of the pens.

When we shared eighteen pens between two people, we halved eighteen so that each person had half of the number of pens.

Explain that when two people are sharing, the number of things is divided in half. If things are shared among three people they have a third each, not a half.

'If you and I share this muffin, we will have half each.'

If the child doesn't seem to be following, you could do some paper folding into halves, or talk about sharing a pizza, cake or apple.

Move on to using larger numbers of items, but always even numbers, sharing between two. Make a note of all the numbers used.

? Can you share these twenty sweets between two people?

? Can you tell me what you are doing as you do it?

Record again for the child, trying to use a good range of the key vocabulary above.

We shared twenty into two groups, so that gave ten in a group.

We divided twenty into two.

Twenty divided between two means they get half each, which is ten.

You could highlight the key words with a coloured pen, or write the key vocabulary on cards.

? Can you use the words 'divided between' in a number sentence?

If the child is finding this hard, take the key words written on cards into the class so that the child can practise each day for a while using the words.

? When we divided between two today, there were other words we could use as well as 'divided'. Can you remember them?

Finish by showing the child the numbers and the divisions they have done.

**? What if we shared eighteen pens among three people?
Would they have half each?**

If the child cannot see that sharing among three would not mean half each, you will need to do some more dividing of even numbers (four, six, eight, and so on) in half – between you and the child or two of the character toys. Emphasise that 'half' is the word we use to describe what each gets when we share equally between two.



'Write a division story using any numbers you choose, but you must divide between two. Now write it again using the same numbers, but try to use completely different words (you can repeat words like 'the', 'and', 'divide', 'two', and so on). Mark any words you need to repeat and we will see who repeated the fewest.'

Spotlight 2

Is not systematic when sharing into equal groups using a 'one for you' approach; does not use the language of division to describe the process

Opportunity for: reasoning about numbers

.....

Treasure hunt

Time 10–20 minutes

Resources

- Eighteen cubes or character toys
- Six rectangles of paper or model cars
- Sentences from previous activities
- Thirty-six 10p coins (or to suit the child)

Key vocabulary

- | | |
|----------------|---------------------------|
| share between | equal groups |
| share into | same number in each group |
| divide between | fair |
| divide into | |

Teaching activity

Explain that today the child is going to solve a problem about a car treasure hunt. Eighteen people want to go on the treasure hunt and six of them want to take their cars. They want to have an equal number of people in each car, so that the teams are fair.

? Can you choose from the things on the table to work out how many people will need to go in each car? Tell me what you are doing as you do it.

If the child finds this too challenging, explain the problem again and suggest what to use to represent the six cars.

? Can you divide the people equally between the six cars so that the teams are fair?

Observe the child sharing to check that they are systematic.

If the child is not sharing systematically, suggest that they need to be more careful. Ask them to count how many there are in each group to make sure it is fair.

If necessary, gather the cubes together and suggest that they share them out again, but more carefully.

? How many people are there in this car now?

? Are there enough for one more person in each car?

? Are all the groups equal? How do you know?

Once you have established that there will be three people in each car, put out the words and sentences from previous activities and ask the child to look for words that they could use to describe what they have done today.

Eighteen people divided into six teams is three people in each team.

We shared eighteen people between six cars and that meant three people in each car.

$$18 \div 6 = 3$$

Eighteen people shared between six cars means that there are three people in each car.

Now extend the activity by saying that one of the teams comes third in the treasure hunt and wins £3.60 in 10p coins. They decide to share it with everyone, all eighteen people (or use an amount of money to suit the child).

? How much money will they get each?

Observe how the child shares the money. Let them do it their own way, sharing between eighteen people, or between the six teams.

If there is time, let the child make their own recording of sharing the money.

Support the child in this sharing, if necessary. Talk about the need to be fair so they must have the same amount of money each.

If they need more practice with sharing money, choose some easy amounts, such as sharing 10p coins between three people.

? What do you think is a key thing to remember for next time?

Spotlight 3

Is not systematic when sharing into equal groups using a 'one for you' approach; does not use the language of division to describe the process

Opportunity for: developing estimation skills

.....

Lots of sweets

Time 10–20 minutes

Resources

- Container with forty-eight sweets or cubes and another with sixty
- Place value (arrow) cards
- Number line
- Two character toys or puppets
- Previous recordings
- Bundles of straws
- Bead strings

Key vocabulary

share between	same number in each group
share into	fair
divide between	half
divide into	halving
equal groups	

Teaching activity

'Today we are going to think about how to share lots of sweets fairly between these two people. This will help you with estimating before you divide the sweets into two groups.'

Show the child the forty-eight sweets without telling them how many there are.



? Can you guess how many sweets there are here?

? Are there more than ten? More than twenty? More than thirty?

Say that we're not going to count them just yet and that the child should think about how they could divide these sweets equally between these two people.

Watch what the child does and build on it.

If they push some sweets to one side and some to the other, you should let them do that and then wait to see if they check that each group has the same number of sweets by counting.

If they start to give them out using the 'one for you' method, you should let them do a few and then point out that they are using a very slow method.

? Can you think of a quicker way of doing that dividing?

Observe what the child does. They might give the sweets out in larger groups.

If the child seems not to have any strategies, you will need to lead them towards the idea of sharing them out in larger numbers than one at a time.



? Do you think there are enough sweets to give the sweets out two at a time? What about five at a time? What about ten at a time?

When the sweets are shared out, support the child in checking each group.

? How can we check that each person has the same number?

If the child doesn't suggest counting them, ask:

? What can we do to find out how many each person has?

Count the sweets with the child. Again, if they count in ones, ask them to think of a quicker way to count.

If the child can't think of a quicker way, suggest counting in twos then putting five of the twos together so you have sets of ten.

Help the child to count: 'ten, twenty and four more in each group makes twenty-four'.

Record what they have done.

Forty-eight divided between
two people is twenty-four
each.

$$48 \div 2 = 24$$

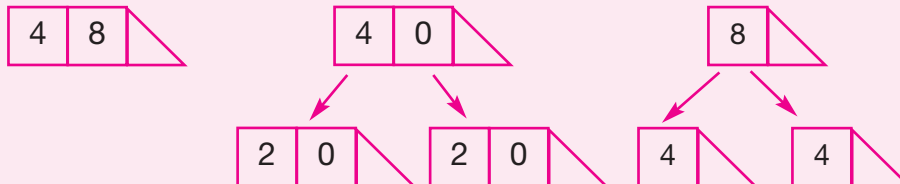
? How many sweets did we start with if we now have two groups of twenty-four?

? How could you find out how many altogether?

Support the child counting on from twenty-four in twos, or putting all of the sweets into groups of ten again: 'ten, twenty, thirty, forty and eight more.'

Put out forty-eight in place value cards and demonstrate the halving and doubling.

Link to finding twenty-four and forty-eight on a 100-square and number line.



You can show the halving with place value cards, if you have time, showing how forty is halved to twenty and eight is halved to four, so half of forty-eight is twenty-four.

It would be helpful to record this on a number line and mark 24 in the middle for the next Spotlight. Model using a bead string.



? So which number comes half way between zero and forty-eight?

If the child cannot see that twenty-four is half-way:

- help them to count along the number line, maybe in twos from zero, establishing that it is 24, then carry on counting in twos to 48;
- show it on the 100-square – from 24 show adding twenty by going vertically two lines, then add four more. (Be aware that children can find it hard to count in ones on a 100-square because they often make a mistake with the starts and ends of lines so end up with the wrong answer!)

? When the people had a fair share each, how many did they have?

Show the child the number sentence you recorded.

‘Twenty-four is half-way along between zero and forty-eight.’

Model using a bead string to fold over, reinforcing the equal parts.

Show the container with sixty sweets.

If you are short of time, just ask the child to estimate how many sweets there are and to talk you through how they might divide them equally between two people. Focus on the need to find a quick way to do it, not sharing them out one at a time.

If there is time to divide the sixty sweets, suggest to the child that they share them any way they want. They could count them first, then share them out; or share them, then count to check that the groups are equal.

If the child starts to do ‘one for you’ stop them and talk them through sharing out maybe two, five or ten at a time.

60 divided equally between
2 is 30 in each group.



? How do you think it helps to estimate how many sweets there are before we share them out?

? What did you learn today about a quicker way to divide?



? If you had a big jar of sweets and you had to share them equally between everyone in the class, what would you do first? Then what? What if there were some sweets left over?

? Of all the ideas people have for how to do it, what might be the quickest way?

Spotlight 4

Is not systematic when sharing into equal groups using a 'one for you' approach; does not use the language of division to describe the process

Opportunity for: recognising relationships between numbers

.....

Dividing with twenty-four

Time 10–20 minutes

Resources

- Two character toys or puppets
- Twenty-four sweets or cubes
- Previous recordings
- 2 cm squared paper

Key vocabulary

- | | |
|----------------|---------------------------|
| share between | equal groups |
| share into | same number in each group |
| divide between | fair |
| divide into | |

Teaching activity

Make a 12×4 rectangle with squared paper. Start with twenty-four sweets in a container. 'Today we are going to do some dividing with these sweets. This is going to help you use words for division, which is important for when you want to share things out to make it fair.'

Remind them about using the forty-eight sweets last time, and show the squared paper with forty-eight squares.

? If we fold this paper with forty-eight squares in half, how many squares will show?

Relate this to the previous number line recording, showing twenty-four as half of forty-eight.

? Do you think twenty-four will divide equally between two with none left over?

If the child can't predict this, remind them about even numbers and how they always share out equally into two groups. Remind them that even numbers end in zero, two, four, six, or eight.

? Is twenty-four an even number? How do you know?

Put out two character toys and observe how the child shares out the twenty-four sweets/cubes equally.

Ask them to tell you what they are doing as they do it.

If the child gets muddled or keeps going with 'one for you' and so on, suggest they give out the sweets in larger groups.

? Are there enough sweets to give them out in tens? Or is that too many at a time? What might be a good number to give out each time?

Encourage the child to make a sensible estimate of how many half of the sweets would be.

**? Can you remember what half of twenty is?
So what might half of twenty-four be? Would it be close to ten?**

This time when you record what they are doing, ask the child to choose some of the words and symbols from previous recordings and you can write these on paper.

divide between
divide into
same number in a group

Support the child in saying number sentences using the words above, for example:

‘Twenty-four divided between two makes two equal groups of twelve.’

‘Twenty-four divided equally into two makes twelve in each group.’

‘If you share out twenty-four equally between two, that makes twelve each, and that is half of twenty-four each.’

? Can you point to the symbol for division in the sentences we wrote last week?

Record the calculation with the division symbol.

$$24 \div 2 = 12$$

? Can you think of another way to read that division sentence?

Finish by asking the child to read to you some of the important words for division.

If you think the child needs more experience with twenty-four, you could divide twenty-four by three in the next session.



? Start at forty-eight and divide between three, so you get to sixteen, then keep dividing between three...

? Keep going. What is the smallest number you can make? (You can use a calculator.)

Spotlight 5: a learning check

Is not systematic when sharing into equal groups using a 'one for you' approach; does not use the language of division to describe the process

Opportunity for: discussing and explaining

Pirate gold

Time 10–15 minutes

Resources

- Cards with the key vocabulary words and any other words for division used by the child
- Yellow or orange cubes
- Number cards: 6, 8, 9, 10, 12, 15
(Resource sheets 1 and 2)
- At least one other child to play

Check: does the child use key vocabulary?

share between	same number in each group
share into	fair
divide between	half
divide into	halving
equal groups	

Teaching activity

'Today we are going to play a game called **Pirate gold**, and we are going to see who wins the most gold. To win gold you must say the division sentences correctly, so this game will help you to be good at talking about division.'

Choose about four or six word cards and read them with the children. Put these cards face up on the table. You can add more cards as the children become more confident.

Note: Emphasise that the amount must be divided *equally* to make it fair.

divided into two

divided into three

divided between two

divided between three

Put the number cards face down on the table and spread them out so that they can be turned over one at a time. When they are turned back, they should be put in the same place as before. Put word cards face up.

How to play

The aim of the game is to win as many pieces of pirate gold (cubes) as possible.

1. Players take turns to turn over a number card and read the number. They then choose one of the word cards (thinking carefully which one is the best for them) and read the whole number sentence. For example:

12

divided between two

'Twelve divided between two makes six in a group.'

The players work out the number sentence using the gold pieces. They must say the sentence correctly – everyone else in the group must listen very carefully to what they say.

2. If everyone agrees that the sentence has been said properly and that the numbers are correct, that player wins all of the pirate gold pieces from one of the answer groups, six in the example given above.

If they say the sentence incorrectly they don't win anything!

They also don't win anything if, when they divide up the gold pieces equally, they find they have gold pieces left over. So they need to think very carefully whether they want to divide by two or by three.

3. The 12 card is turned back carefully so that it stays in the same place on the table.

Warn the players that they will find it helps them if they remember where that 12 is – and any of the other numbers displayed.

The aim of the game is to win as many pieces of pirate gold as they can, and players will find that some numbers are more useful than others.

If a child is finding the game hard, support them when they choose a suitable card to divide between, and also as they use cubes to work out how many gold pieces in each group. For example:

? How many groups do you have to divide the twelve between?

? Have you got two equal groups? How do you know?

After a few turns you could ask the children to stop and reflect on what is happening.

? To win more gold, which are the best number cards to use? Those with larger or smaller numbers?

Establish that they will win more gold with twelve divided into two than six divided into two.

? When Dan said his sentence, do you think he said it correctly? What else could he have said?

? Ali, can you say the sentence that Dan just said?

? When Lisa said 'shared between two' was that right?

Variation

- It can turn into more of a game of chance if the number cards and the word cards are put into two shuffled piles and players take the top one of each pile and then return that card to the bottom of the pile. Doing this, some of the divisions won't work out without having a remainder, so the player will win nothing.

Learning outcomes

By the end of this set of activities, children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- be systematic when sharing into equal groups using a 'one for you' approach;
- begin to have more efficient strategies for sharing, such as sharing out two or more at a time;
- begin to remember some division facts;
- use the language of division to describe the process;
- begin to show some understanding of estimating before dividing;
- check calculations by counting in increasingly efficient steps.

Does not understand that 'sets of' or 'groups of' need to be subtracted to solve the problem

Opportunity for: developing mathematical language

Resources

- Cubes
- Bead string
- Wipe-clean number lines
- Calculator

Key vocabulary

- | | |
|-------------|-----------|
| divided by | pair |
| hops of two | sets of |
| subtract | groups of |

Teaching activity

Time 15–20 minutes

Explain to the child that today's activity will help them to divide numbers into equal groups, but that they are going to do it on a number line by drawing hops of the same size.

'We will be taking away equal groups each time because, when we are dividing, we must make equal groups of things.'

? Can you tell me what you think putting things into equal groups means?

Encourage the child to choose anything to show you what they think 'equal groups' means. Build on what the child says.

If the child struggles with equal groups, you can support them by helping them to count to check that each group has the same number of cubes.

? How many cubes has this group got? So how many does this group need? Are all the groups the same size? How many equal groups did you make?

Congratulate the child on any equal groups they made by sharing. Now explain that they are going to make equal groups by taking away along a number line and with a bead string. This is another way of dividing and they are going to be saying '*divided by*' when they use these bead strings and number lines.

Look at a bead string of ten beads with the child. Count them in twos with the child to make sure there are ten.



'Look how we can take away an equal group of two at a time. Subtract two, then another two, then another two.'



? Can you take away another two by moving the beads?

? How many more groups of two can we make?

If the child seems confused, put the beads back at the right-hand end of the string and ask the child to move them two at a time to the other end, saying what they are doing. Support the child through that process.

‘There are ten beads. Taking away two means now there are eight left. Take another two so there are six left, and another two and another two, and another two.’

? How many sets of two did we subtract? So how many groups of two are there in ten?

? Can you say a sentence about what we did about subtracting two at a time from ten?

Note the words that the child is able to use and those that need to be taught (see key vocabulary above), for example: ‘we hopped back in twos from ten and we did five hops, or we divided ten by two and made five equal groups.’ (Put emphasis on *divided by*.)

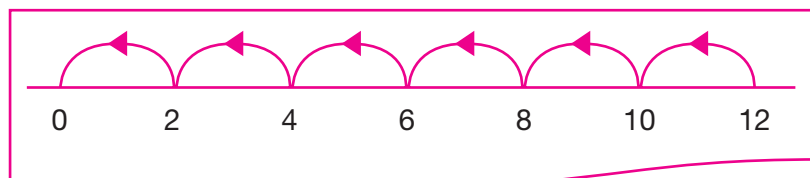
Put two more beads on the bead string, so you are working with twelve.

? How many beads are there now? Do you think we will be able to make equal groups of two? Can you move them two at a time, telling me what you are doing?

Support the child as they talk through what they are doing, helping them to use some of the key vocabulary above, such as:

‘We are starting with twelve beads and we are dividing them by two, so we take away a group of two each time. Now we are left with ten beads and take away another two so there are eight left ... and so on.’

Record the process on a number line.



? Can you talk me through what we did?

The child’s answer should be similar to the following:

‘We started at 12 and we hopped back two to get to 10, another hop of two got us to 8, then another hop of two and we were on 6. Then, taking away another two we were on 4, so we only needed two more hops, a hop to get to 2 and then another and we were on 0 and we stopped there.’

? Can you count how many hops we made? So how many hops back of two are there when you start on 12?

? How do you think we could write this down as a subtraction?

If the child can’t cope with this, you could say that you are going to make a very long ‘taking away’ number sentence.

$$12 - 2 - 2 - 2 - 2 - 2 - 2 - 2 = 0$$

? How many sets of two did we take away?

? So can we say there are six groups of two in twelve?

We can say, twelve divided by two is six. When we divide twelve by two, we end up with six sets of two.

Record this for the child.

$$12 \div 2 = 6$$

? What number did we divide twelve by? (2)

If the child needs more practice, repeat with a bead string of twelve and take away three each time, establishing that there are four groups of three in twelve.

Show the calculator to the child.

? Which two operations are we using today? Can you show me the correct keys?

Stress that when making equal groups you use the division sign, but you can also use the subtraction or take away sign.

? Are you surprised by that? Why?

Spotlight 1

Does not understand that 'sets of' or 'groups of' need to be subtracted to solve the problem

Opportunity for: solving real-life problems

Snowy day

Time 15–20 minutes

Resources

- Sixteen blue and eighteen black cubes
- Number lines

Key vocabulary

ten, twenty, thirty, etc.	hop back/forward
backwards	subtract
forwards	pair
divided by	sets of
hops of two	groups of

Teaching activity

Explain to the child that today they are going to do some dividing by taking away. They are going to hop back on a number line and this will help them to learn more about dividing by different numbers. They are going to start by dividing by two.

'Pretend your class is away at an activity centre. Pretend that it is a snowy day, so the teacher tells you to count the gloves that have been in the lost property box for a long time, so that people can wear gloves to go out.'

Give the child the sixteen blue cubes and ask them to imagine that the cubes are gloves.

? How many gloves are there? How many children could have two gloves? How could we work that out?

Listen to what the child does and says to solve the problem.

Follow on from what the child says, noting down the words the child uses for division and subtraction.

? What was the picture in your head as you worked that out?

If the child needs support here, suggest that they put the cubes in pairs, like pairs of gloves and then count them.

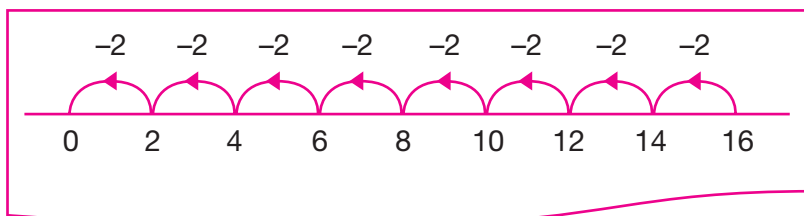
'One pair for one child. Another pair for the next child...'

? How many pairs do we have? So how many children can have gloves?

'Let's do this calculation on a number line. Let's mark 16 here. Now we are going to hop back in twos.'

If the child is unsure, show them how to do a hop back of two, getting to 14.

You might also find it helpful to put the pairs of cubes into one long line, breaking one pair off as the child does the hop back of two each time.



Relate this to a pair of the cubes and also to the repeated subtraction calculation.

$$16 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 = 0$$

? How many are left now? So can you do another hop back in twos?

Keep counting back in twos, taking off the pairs of cubes/gloves as you go.

? How many children can have a pair of gloves?

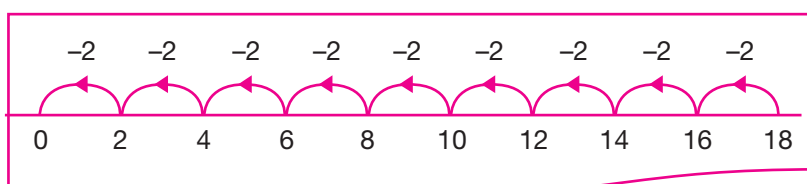
? So what is sixteen divided by two?

For some children it might then be best to choose a smaller number of cubes for the next activity, for example, ten or twelve rather than eighteen.

For children who are ready to move on, you could go back to the snowy day and say that the teacher at the activity centre has found eighteen Wellington boots.

? How many pairs is that? So how many children can have a pair of boots (if they fit!)?

Use another number line and ask the child to mark 18 and hop back in twos.



They might need support to do the related repeated subtraction.

$$18 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 = 0$$

‘We write the eighteen, then we keep counting back in the number we are dividing by.’

? What is the number we are dividing by?

(Keep these repeated subtractions and number line hops to refer to in other sessions.)

Show a formal recording of eighteen divided by two.

$$18 \div 2 = 9$$

‘So nine people could each have a pair of Wellington boots.’

Finish by asking the child if they can write the sign we use when we subtract and the sign we use for division.

If the child is not recognising signs, some free or structured play with a calculator can be helpful.

Spotlight 2

Does not understand that 'sets of' or 'groups of' need to be subtracted to solve the problem

Opportunity for: solving real-life problems

Chocolate bars

Time 15–20 minutes

Resources

- Wipe-clean number line
- Variety of chocolate bars (paper copies of the bars are useful, or double wrap the bars in foil)
- Cubes

Key vocabulary

divided by	pair
hops of two	sets of
subtract	groups of

Teaching activity

Explain that today we are going to do some problems involving dividing chocolate and money, and that we are going to work out the problems on a number line. We have to think carefully about the words that we use to do division calculations, so we are going to write down some of the words used.

? Can you remember doing subtractions on a number line to work out some division calculations?

Show one or more of the number lines with the hops back from a previous session.

? Can you remember what we did with this number line?

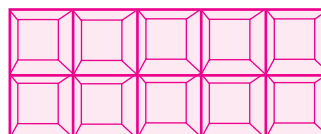
If the child can't remember, talk through what you did. 'We started at 16 and we kept hopping back in twos: 14, 12, 10, 8, 6, 4, 2 and 0.'

? How many hops of two did we make?

Note down any words for subtraction and division that the child uses.

Talk the child through counting forwards in twos on the number line, showing how there are the same number of hops whichever way you go.

Show one of the bars of chocolate to be divided by two.



Talk about the bar and ask the child to count the chunks.

? How many children can have two chunks of chocolate?

Show the division you will be doing.

$$10 \div 2 =$$

'Ten chunks divided by two.'

If the child needs quite a bit of support, and you intend to eat the chocolate, you might find it helpful for the child to make a copy of the bar in cubes.

? Can you break off two chunks for the first child?

Support the child using a range of language as they do the activity (see key vocabulary above).

'If I break off two chunks, that leaves me with eight chunks. Then I can break off another two, leaving me with six chunks ... and so on.'

? So how many children can have two chunks?

You or the child should record what you are doing on a number line. Mark the 10 first. Then you or the child should draw hops of two, counting back from 10 as the chunks (cubes) are broken off in twos.

If the child seems anxious about this, you could start the process and leave one hop almost done so that they can finish it off.



'Starting at ten and breaking off two chunks leaves us with eight. Break off another two and do another hop back of two.'

? How many chunks are left now? Where are we up to on the number line?

Carefully record each stage and talk the child through them, encouraging them to join in to describe the process in their own words. Note down the words they use. (You can use this list to see which words they are not using in the key vocabulary above.)

? So how many chunks of two do we have?

How many children can have two chunks each?

? What is ten divided by two?

? Do you think it would work if we hop forwards rather than backwards?

If the child seems to be understanding, you can make a start on a repeated subtraction and ask them to finish it, letting them talk it through with you as they finish it off.

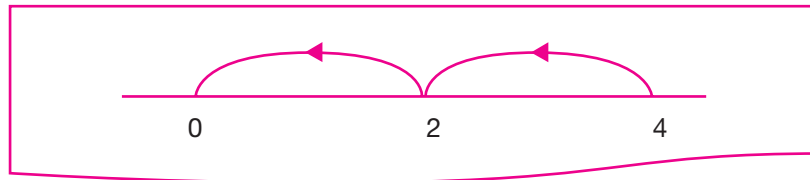
$$10 - 2 - 2 - 2 - 2 - 2 = 0$$

If the child needs more help with counting back on a number line, use a chocolate bar with four, six or eight chunks.

Four divided by two

? How many children could have one chunk?

? How many children could have two chunks?

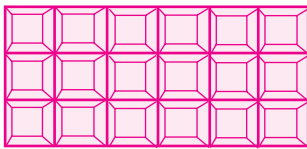


Extend the activity with larger numbers.

Show another chocolate bar with eighteen chunks.

Let the child count the chunks and make a cube copy.

? How many children could have three chunks of chocolate?

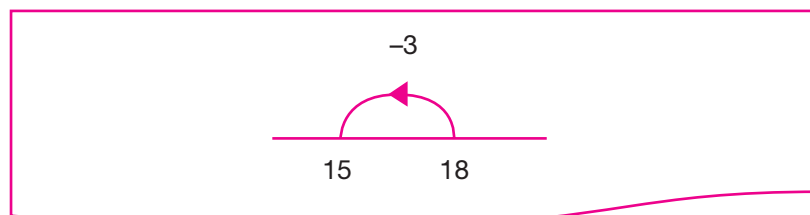


Eighteen divided by three

? Can you draw the number line which we need to work this out? Which number goes at the right-hand end? Can you write the eighteen?

? We are doing eighteen divided by three, so how big do you think the hops back need to be?
(Three)

Encourage the child to do the recording, hopping back in steps of three.



Let the child do as much of the working out as they can, by hopping back in threes and breaking off cubes in threes.

? Can you write down the repeated subtraction that is another way of showing your hops back in threes?

18 - 3 - 3 ...

? How many children could have three chunks of chocolate? So what is the answer to eighteen divided by three?

? Do you see what I mean about having to be careful about the words that we use?
Can you tell me some of those words?

Spotlight 3

Does not understand that 'sets of' or 'groups of' need to be subtracted to solve the problem

Opportunity for: investigating numbers

7 Y2 X / ÷

Is there another way?

Time 10–15 minutes

Resources

- Cubes
- Number cards 0–10 (Resource sheet 1)
- Paper and at least four different coloured pens
- Some of the vocabulary written on card/paper

Key vocabulary

- | | |
|-------------|-----------|
| divided by | pair |
| hops of two | sets of |
| subtract | groups of |

Teaching activity

Explain that today we are going to do some work with twelve cubes and counting backwards and forwards on a number line. We will be using some special division words again today.

Ask the child to count out twelve cubes and make them into one long 'train'.

Now explain that we are going to take away cubes to show twelve divided by two. (Emphasise divided by 2.) That is the number of cubes to be broken off each time, and the size of the hops along the number line.

Ask the child to put a number 2 card onto the right place on the calculation and to read the calculation to you.

12 divided by 2

? How many cubes are we going to break off each time if we are dividing by two? (Two)

? When you have taken two off, how many are left? (Ten)

Mark 12 on a piece of paper, making the line about the same length as the 'train' of twelve cubes.

? What size of hops are we using if we are dividing by two? (Two)

Can you draw a hop back of two with the green pen?



? How many cubes are left on the cube 'train'? (Ten)

? What number are we down to on the number line? (10)

? What number are we dividing by? (Two)

So how many should you break off this time? (Two)

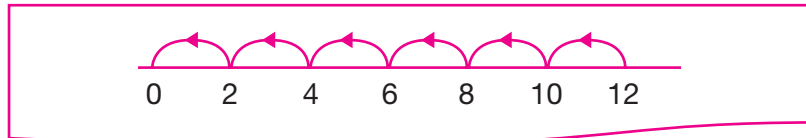
As each pair of cubes is taken off, record the step of two on the number line and count how many cubes are left on the cube 'train'.

Keep the broken-off pairs of cubes as towers of two.

If the child counts the cubes in the cube 'train' one at a time, encourage them to count back from the last number, so twelve take away two is ten. If the child is struggling with this, jot down two, four, six, eight, ten, twelve on a bit of paper and remind them of counting in twos, forwards and backwards.

Keep repeating: 'We are dividing by two so we break off two each time.'

- ? How many cubes are left on the cube 'train'?**
What number are we down to on the number line?



- ? How many hops of two did we make when we divided twelve by two? (Six)**
? What is the answer to twelve divided by two? (Six)
? Can you say the answer in a number sentence, using the words divided by two?

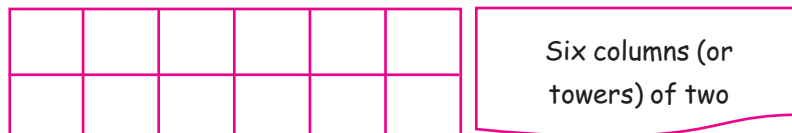
Record for the child:

Twelve divided by two makes six equal groups of two.

- ? Could you say that number sentence in a different way?**

For twelve divided by two you make six equal hops of two.

Ask the child to make the broken-off pairs of cubes (kept as six towers) into an array.



- ? Can you tell me how many rows and columns there are in this array? How many cubes are there altogether?**

Establish that there are six columns and two rows.

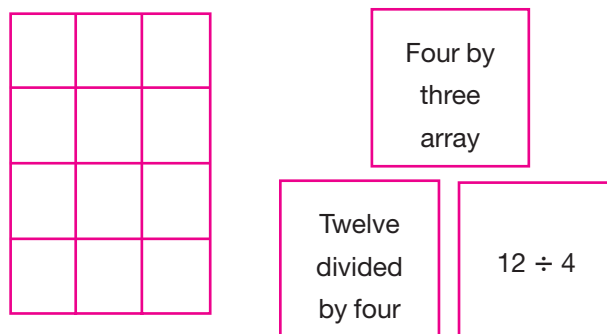
- ? If you shut your eyes, can you see the cubes in your head, arranged in six towers or columns of two?**

- ? Can you use those same twelve cubes to make a four by three array?**

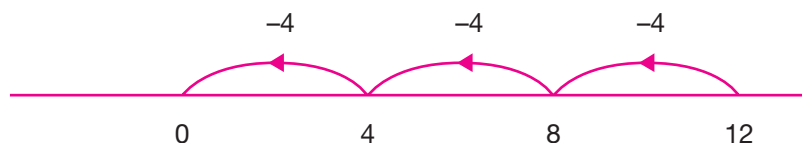
If the child finds that hard, help them to put together a four by three array. Establish that there are still twelve cubes, but that we have arranged them differently.

- ? What size hops do we do when we are dividing by four?**
Can you draw those hops in blue?

If the child needs help with that question, lead them towards breaking the array up by taking four off each time.



On the number line they should start at 12 and land on 8, 4 and 0, making three hops of four.



Establish that the number which you are dividing by is both the size of the hop back and the number of cubes you break off each time.

If you have time, you could work with the child to make other possible arrangements of twelve cubes. With a red pen, you could do four hops of three, or twelve hops of one, or one hop of twelve, or two hops of six, or six hops of two.

? Can you think of another way to arrange the twelve cubes as an array or rectangle?

? Which other numbers are like twelve, so that you can make several different sizes of hops going from the number to zero each time?

Spotlight 4

Does not understand that 'sets of' or 'groups of' need to be subtracted to solve the problem

Opportunity for: developing mental images

.....

Ribbons, puppets and pennies

Time 10–20 minutes

Resources

- Cubes
- Two metres of ribbon
- Metre stick
- String and scissors
- Coins
- Buns or circles of paper
- Finger puppets
- Two sets of number cards 0 to 40 or 50 (Resource sheets 1, 2, 3 and 4)
- Some of the number line drawings from previous activities

Key vocabulary

divided by	pair
hops of two	sets of
subtract	groups of

Teaching activity

Explain that today we are going to do some dividing using a number line and that they can use any of the things on the table to help them.

You could show some of the previous number line drawings.

Write an 'empty' number sentence for the child to use for each problem, putting in number cards in appropriate places.

	divided by		makes		equal groups of	
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? What can you tell me about dividing using a number line?

Show the child a bit of ribbon that is 40 cm long. Put the ribbon next to the metre stick.

? If we divided this ribbon into 10 cm pieces, how many pieces would we cut it into?

Establish that to divide by ten you would cut it into four pieces.

Ask the child to shut their eyes and try to see a picture in their mind of the ribbon divided into four pieces.

If you think that the child needs to do some folding and cutting, cut paper strips or string and model the activity above. Create examples based on 20 cm or 30 cm if you think more examples would be helpful.

Now tell the child that you are going to ask them some questions and that they should decide how they are going to answer them.

'Some children are writing plays to perform in a puppet theatre. Their teacher has a total of eight finger puppets.'

? How many children can have one finger puppet on each hand?

? How will you work that out?

If the child seems confused, repeat the question. If they still don't know how to work it out, suggest that they get some cubes (ideally ones that will fit onto a finger like a finger puppet does) and also take a pen and paper and draw a number line with eight at the end.

Clarify that they are dividing eight by two.

? When we divide by two, how many do we need to take off each time?

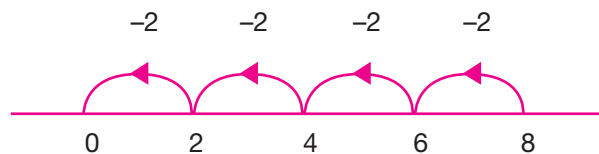
Ask the child to shut their eyes and try to see the eight finger puppets in their head.

? Can you see a number line to eight in your head?

? We are dividing by two, so what does that tell you we need to do? (*Make hops of two on the number line and/or put the cubes in pairs.*)

If the child needs support to do this, help by making the hops back along the number line, counting back in twos.

Link this to moving two of the cubes (see *Spotlight 3*). 'We have eight cubes, so we need to take away two of them for the first pair of puppets.'



? How many hops of two do we make when we divide eight by two? What is the answer?

Help the child to make a sentence explaining what they have done, using the key vocabulary.

We had eight finger puppets and we divided them by two and made four equal sets of two. So four children can have two finger puppets each.

Select from the five problems below.

They can all be solved by repeated subtraction. For each problem, write the calculation (or fill in the 'empty' number sentence above) and count on or back on a number line.

You can change any of the numbers to suit the child. (The only one with a remainder is for the whole class to solve.) Let them choose freely from mathematics equipment to help them solve the problems.

Each time ask:

? How did you work that out? Was there a different way of doing it? Can you see the number line in your head?

? 1. Sally has sixty pence. How many 10p sherbet dips can she buy?

$60 \div 10 = 6$ 'Sixty divided by ten is six, so Sally can buy six.'

Some children might solve this quickly mentally. Praise them for that, and ask them how they did it.

Many children find it hard to explain their methods, but if they have been having problems with the number line hops, you could draw this one for them to show them what they might have done in their head – but they did it so quickly they couldn't 'see' it! Start at either 0 or 60 and count in tens to make six steps of ten.

? 2. Tim's hens have laid twenty-four eggs. How many boxes of six can he fill?

$24 \div 6 = 4$ 'Twenty-four divided by six is four, so Tim can make four full boxes.'

? 3. There are twenty-six slices of bread in a loaf. How many people can have two slices each?

$26 \div 2 = 13$ 'Twenty-six divided by two is thirteen, so thirteen people can have two slices each.'

? 4. There are thirty-two strawberries in a punnet. How many people can have eight each?

$32 \div 8 = 4$ 'Thirty-two divided by eight is four, so four people can have eight each.'

? 5. Abdul is making five clay bugs. He wants to use drinking straws for legs. Each bug needs six legs. He thinks he can cut straws in half and they will be about the right length. How many straws will he need? (15)

? How did you work that out?

? Was that a hard question? Why?

For question 5, it might be useful to ask children to record in their own way how they worked it out. It is a two-stage question and children might have added six five times, then halved the result. But they might have many different ways of solving it.



? Mrs Bun the baker packs her sticky buns in fours. If she makes forty buns, how many full packs can she make?

? One day she found that she had three buns left over after packing them. How many buns might she have made that day?

? Write a story problem for thirty divided by five. Make a number line hopping picture for your story. Did you use counting back or counting forward? Can you use either?

Spotlight 5: a learning check

Does not understand that 'sets of' or 'groups of' need to be subtracted to solve the problem

Opportunity for: discussing and explaining

7 Y2 X / ÷

Stick break

Time 10–20 minutes

Resources

- Cards with 'divide by two', 'divide by three', 'divide by ten' written on them
- Cube 'trains' of eight, nine, ten, twelve, fourteen, fifteen, sixteen, eighteen, twenty, thirty
- Some extra 'ten trains'
- Sticky tape
- Numbered number line to 30
- 100-square
- At least one other child to play the game

Check: does the child use key vocabulary?

divided by	pair
hops of two	sets of
subtract	groups of

Teaching activity

Explain that today they are going to play a game called **Stick break**. Show them the cube 'trains' and say that they are the 'sticks'. Explain that we will also use the following cards.

divide by two

divide by three

divide by ten

The aim of the game is to end up with no cubes at all.

Put the sticks on the table and label each one with the number of cubes. Put the cards face up on the table.

How to play

1. Players take turns to choose a stick and a number card, for example, the twenty stick and the 'divide by ten' card.
2. The players then need to work out how many sticks to break off the long stick so that there will be no cubes left. They must break up the stick and say what they are doing in a number sentence, reading the card they chose. For example: 'Twenty divided by ten makes two equal sets of ten with none left over.'
3. If everyone agrees that they have said their number sentence correctly and have worked out the numbers correctly, they win no cubes – which is a good thing! They need to try not to win any cubes! If they do win cubes they are likely to lose!
4. The stick is put back together again, and the card and stick remain on the table for others to use.
5. If a player chooses a stick and a card where they do get a remainder, they will win ten cubes. For example, if a player chose the eighteen stick and the divide by ten card, they would not be able to make equal sets of ten.

If children find the game hard, support them with counting in twos, threes and tens, with either a number line or 100-square to help.

Prompt children to listen very carefully.

When all the children have had a few turns, ask them to stop and reflect on what they have done.

? Which sticks are good to divide by two?

? Why did you choose the divide by ten card for that stick?

Variation

- Play the game with piles of coins, for example, fifty pence, thirty pence, one pound, eighteen pence, forty-five pence, seventy-five pence, and so on.

? Do we need to make some more 'divide by' cards? Which numbers do we need?

Learning outcomes

By the end of this set of activities, children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- understand and use a range of vocabulary for division;
- recognise that when *divide by* is used, the problem can be solved by repeated subtraction;
- form mental images of arrays and number lines;
- begin to learn some division facts.

Is not confident in recalling multiplication facts

Opportunity for: exploring mathematical language

Resources

- Multiplication grid 1 (Resource sheet 35) or for Year 6 work use Multiplication grid 2 (Resource sheet 36)
- Different coloured pens, pencils
- Squared paper
- Cubes, counters
- Bead strings

Key vocabulary

groups of	equal groups
lots of	sets of
multiplied by	rows of
array	columns
divided by	

Teaching activity

Time 15–20 minutes

Explain to the child that today's activity will help them to improve at multiplying and remembering their multiplication tables. Show them the equipment.

'You can choose any of this equipment to show something that you know about multiplication.'

Listen carefully to what the child says and build on any relevant words they use or images they draw.

If the child is unsure what to do, give a little more guidance.

? Can you make me a picture on paper to show six groups of two?

? Or can you use cubes, counters or bead strings?

Help the child to make equal groups.



? Can you tell me about this arrangement of cubes?

Establish that there are six groups and two in each group.

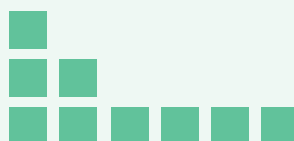
? Can you put these cubes in an array to show two multiplied by six?



- Two multiplied by six
- 2×6
- Six groups of two makes twelve altogether
- $2 + 2 + 2 + 2 + 2 + 2$

Once the child has some kind of array, or a set of equal groups, ask them to talk about it.

If the child seems uncertain, you might want to probe for any misunderstandings by making a random arrangement of cubes, for example:



? Is this an array to show multiplication and division facts? Why not?

Establish that multiplication and division are about *equal* groups.

Note the language of multiplication that the child uses, and any errors.

Help the child to count the columns in twos and to record two multiplied by six in any way they want.

If the child has difficulty with the language, or with understanding that the array is two multiplied by six, you could encourage the child with some arm actions to help them to remember the way to record an array.



Point your left arm away from you, parallel to the column of two cubes and say 'two'.

Then, using your right arm, move it away from you, jumping it along the columns of two cubes as you say, 'multiplied by six'.



? If that is two multiplied by six, what would two multiplied by five look like?

Establish that you could take away one group of two, or let the child draw or make this array with cubes or counters.

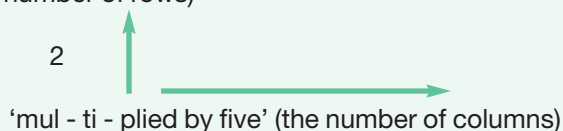
? Tell me about this array.



- Two multiplied by five
- 2×5
- Five groups of two makes ten altogether

If the child needs more support, use the hand movements again.

'Two going this way' (the number of rows)



'So that is two multiplied by five.'

$$2 + 2 + 2 + 2 + 2$$

$$2 \times 5$$

Repeat with any other arrays and groups of two. Eventually end up with an array of 2×10 and ask the child to use the array to count.

? How many cubes do we have in this array altogether?

If the child starts to count in ones, stop them.

? Is there a quicker way to count?

If they seem unclear about what to do, help them to count in twos.

If they count in tens, accept this but point out that they can also count in twos and with two multiplied by ten it is two that is the group size. So, although you get the same answer as $10 + 10$, really this calculation is $2 + 2 + 2$, etc. But if you look sideways or move it a quarter turn, it is $10 + 10$.

Keep the ten groups of two cubes on the table.

? How far can you go counting in twos?

Using *Multiplication grid 1* (Resource sheet 35) (or *Multiplication grid 2* (Resource sheet 36) for Year 6) ask the child to fill in the vertical column under the 2 in pencil. Establish that two multiplied by zero is zero, and that two multiplied by one is two and so on.

If the child is struggling with this, use the array of twenty cubes or counters to count along the columns in twos.

Mark with a blue dot in the relevant space any facts that the child takes a while to remember or work out, and put a red dot next to any which they are not able to work out or remember at all.

‘I’m just putting a blue mark beside the ones you have to stop and think about. We can practise these over the next few weeks. The red dots are ones we can work on together.’

Note: Many children are likely to feel that they have failed at learning multiplication tables, and the resource sheet is designed to help both you and them to see how many facts they do know and also identify which they need to work on. To focus on success you can:

- record the number of facts known at the bottom of the sheet and date it;
- then work on other facts, perhaps counting how many blue and red dot facts still need to be learned and going over these in green pen once they are learned;
- help children to see that there are not dozens of unrelated facts to learn;
- keep praising and reassuring, using games, for example, to help with consolidation, as the number of unknown facts diminishes over time.

If you have time, end by asking the child to chant the two times table, using the language:

One two is two.
Two twos are four.
Three twos are six.
Four twos are eight...

This will link closely to division questions, such as ‘How many twos make eight?’

Help children to make the link to division by sometimes chanting division tables, for example:

Two divided by two is one.
Four divided by two is two.
Six divided by two is three...



? **What is two multiplied by zero? What is two hundred multiplied by zero? What is two million multiplied by zero?** (*Any number multiplied by zero is zero.*)

? **Let’s chant the two times table together. If we went on and on counting in twos like this, would we eventually say fifty-seven? Why not?** (*It is an odd number.*)

? **If you count in twos and say two as the first number, what would be the fiftieth number you said? (100) So what would be the fifty-seventh number? (114)**

Spotlight 1

Is not confident in recalling multiplication facts

Opportunity for: exploring number patterns

Double twos

Time 10 minutes

Resources

- *Multiplication grid 1* (Resource sheet 35) or for Year 6 work use *Multiplication grid 2* (Resource sheet 36)
- L-shaped card
- Different coloured pens, pencils, paper
- Bead strings
- Cubes
- Dominoes

Key vocabulary

groups of
lots of
multiplied by
double and double again

Teaching activity

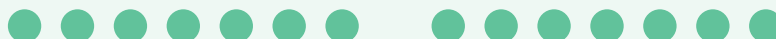
'Today we are going to do some doubling and we are going to use doubling to find ways to remember the two and four times tables.'

Put out the dominoes and ask the child to find the doubles.

? **How did you know that domino was a double?**

? **What is double three? What is double six?**

If the child needs more help with understanding what doubling means, set out some cubes (or use a bead string) and then double the number. For example, lay out four cubes and another four, five and another five, six cubes and another six, then ask the child to put out cubes or beads to show double seven.



Explain that double seven is fourteen.

To make the link from doubling to the two times table, record for the child the numbers one to ten and their doubles, making the link from this to the numbers in the two times table which the child wrote last time on their *Multiplication grid 1* (Resource sheet 35).

Two times table

One doubled is two
Two doubled is four
Three doubled is six
Four doubled is eight

Show a doubling method for working out the four times table.

'One doubled is two and then if we double the two we get to four. Two doubled is four and if we double the four we get eight.'

Two times table

One doubled is two
Two doubled is four
Three doubled is six
Four doubled is eight
⋮

Four times table

Two doubled is four
Four doubled is eight
Six doubled is twelve
Eight doubled is sixteen
⋮

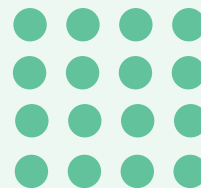
If the child is finding this too challenging, you can make the doubles with cubes or on a bead string as above. Explain that four doubled is eight and then if you double the eight you get to sixteen, illustrating the number relationships using the equipment.



4



8



16

Help the child to fill in the four column on *Multiplication grid 1* (Resource sheet 35), reminding them that four multiplied by zero is zero, if necessary.

? What number shall we stop at? (Try to get to 4×10 .)

? So, if we don't get to forty, what might that tell us? What if we get to thirty-nine?

Explain landing on 40 as a checking method and that we can sometimes make little mistakes so we must always find ways to check what we are doing.

Now, or in a later session, you might want to explore the patterns in the last digit of both the two times and the four times tables.

? What pattern can you see in the last digit of each of our numbers?

? Why do you think this happens?

Say that now we are going to circle every other number in the two times table. Show how every other number is part of the four times table.

0 4 8 12 16 20

Help the child to chant the four times table:

'One four is four.
Two fours are eight.
Three fours are twelve...'

If the child finds this hard, talk them through some strategies which they could use, for example counting on in fours.

Use a bead string to make groups of four.



'One four is four, two fours are eight, then add another four to make twelve.'

Help the child to see the pattern in the final digit. Remind them to check their numbers and to say the ending digits in order again.

If the child struggles with writing the multiples of four, encourage them to go as far as they can and to say the end digit pattern to you again. Look again at how the four times table can be made by doubling the two times table.

? What is double five? So what is five multiplied by two?

? How can we use five multiplied by two to find five multiplied by four? ($5 \times 2 = 10$, so 5×4 is double 10)

? If you were writing the answers to the four times table and you wrote down seventeen, how would you know that you had gone wrong somewhere?

? Look at your multiplication grid and show me how you can find the answer to four multiplied by seven.

Make sure that the child can use the multiplication grid, if necessary making them an L shape with card which they can line up with the 4 at the top of the resource sheet and the 7 at the side to read off 28 in the relevant square.

×	1	2	3	4
0	0	0		0
1	1	2		4
2	2	4		8
3	3	6		12
4	4	8		16
5	5	10		20
6	6	12		24
7	7	14		28

Note: When children are working in class using multiplication to solve problems, you might find that an L-shaped card and the completed multiplication grid can help children to have experience of engaging with the problems without feeling defeated because they can't remember their multiplication facts.

Using an L-shaped card helps to prevent mistakes and can help dyslexic children and others.



? Which other multiplication tables might you be able to work out, starting by doubling the twos?

? How could you use doubling to work out the three and six times tables?

? Can you work out the nine times table with doubling? (Yes, if you work out a 4.5 times table! No, if you don't.)

Spotlight 2

Is not confident in recalling multiplication facts

Opportunity for: developing mental images

Fives and tens

Time 15–20 minutes

Resources

- *Multiplication grid 1* (Resource sheet 35) or for Year 6 work use *Multiplication grid 2* (Resource sheet 36)
- L-shaped card
- Different coloured pens, pencils, paper
- 100-square
- Base 10 equipment
- Number cards 1–10 (Resource sheet 1)
- Cubes
- Place value (arrow) cards

Key vocabulary

groups of
lots of
multiplied by
array

Teaching activity

‘Today we are going to do some work on the five and ten times tables.’

? Tell me what you already know about the ten times table.

? Can you circle the numbers in the ten times table on the 100-square?

If children find this too challenging, help them to find the tens numbers, talk about the zero endings, and lay out some numbers with Base 10 equipment.

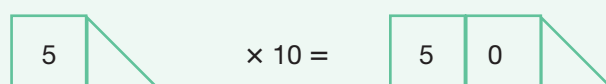
? How many ten rods do you need to show $10 \times 5 = 50$?

Help the child to fill in the multiples of ten (10, 20, 30, etc.) on *Multiplication grid 1* (Resource sheet 35).

If the child needs more support, show some 1 to 10 number cards, asking the child to multiply ten by each number. For example, show five and the child needs to say ‘ten multiplied by five is fifty.’

Support the child to see that the five moves one place to the left, rather than saying *you add a nought*.

If the child doesn’t see the pattern, you might find that place value cards help to show how five moves left into the tens column and there are no units.



Help the child to find the fives on the 100-square and fill in the numbers on the multiplication grid.

If the child needs help to visualise the five times table, you could set it out in cubes in columns, or in a dice pattern.

$$5 \times 1 = 5$$



$$5 \times 2 = 10$$



$$5 \times 3 = 15$$



? When you wrote the numbers on the multiplication grid, which answers were already there in other multiplication tables? (For example, 4×5 was there in the four times table.)

? Why do you think that some of the numbers in the five times table are the same as the numbers in the ten times table?

If the child cannot see that the answers in the five times table are half those in the ten times table, you will need to demonstrate this with cubes or any other equipment which the child finds helpful.

? What do answers in the five times table end in?

? What do answers in the ten times table end in?

With shuffled number cards, show one card at a time and ask the child to multiply five by the number. For example, show seven and the child must say 'Five multiplied by seven is thirty-five', or they could say 'Ten multiplied by seven is seventy and half of seventy is thirty-five'.

Select a few 'blue dot' numbers from the child's resource sheet (multiplication facts that the child can work out but doesn't know by heart). You could write just a few of these on paper for the child to take back to class.

Explain that you are giving the child some multiplication facts that they nearly know by heart. They are going to focus on these this week to see if they can learn them by heart. You could also set them the challenge of spotting these facts in any lessons during the week.

? What did you do today that you found easy? What was hard? What are you good at?



? Which do you find are the hardest multiplication facts to learn? Why are they hard?

? Can you think of a way to help yourself to remember these facts?

Spotlight 3

Is not confident in recalling multiplication facts

Opportunity for: exploring mental images

Lots of ways

Time 10–15 minutes

Resources

- *Multiplication grid 1* (Resource sheet 35) or for Year 6 work use *Multiplication grid 2* (Resource sheet 36)
- L-shaped card
- Different coloured pens, pencils, paper
- Cubes, counters
- Number lines
- Bead strings

Key vocabulary

- | | |
|------------|---------------|
| groups of | adding |
| lots of | multiplied by |
| sets of | array |
| rows of | hops of three |
| columns of | |

Teaching activity

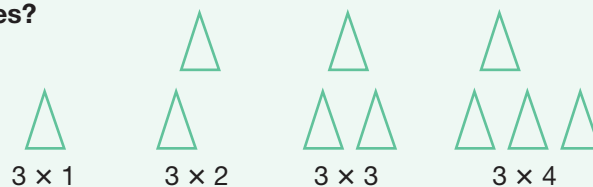
‘Today we are going to work on the three times table.’

? How far can you count in threes?

? What number should you get to for ten threes?

If the child needs help counting in threes, you might want to use triangles or cubes in rows of three.

? How many sides?



Look at the child's multiplication grid (Resource sheet 35).

? Which multiplications in the three times table are already on the grid in different tables? (For example, 5×3 and 10×3 .)

Keep helping the child to see that remembering all the multiplication facts is a manageable task because there are so many repeats, and because there are patterns in the endings of the numbers (though the endings in the three times table are not as straightforward as some other tables).

Help the child to identify any number facts that they already know. You could put a green tick over any blue dot and red dot numbers that the child now knows, or let the child go over the numbers in green pen. You could also put an encouraging note at the bottom of the page and date it.

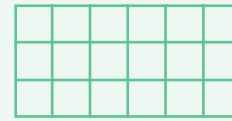
? Choose a number to multiply three by and show it with your favourite equipment.

Build on what the child says and note down any strong points or difficulties which they demonstrate.

If the child can't do this activity, move to the following 'three multiplied by six' activity.

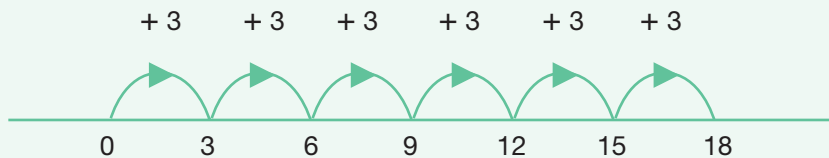
? Do you have a picture in your head when I ask you something like 'What is three multiplied by six?'

Help them to draw pictures or make arrays or hops along a number line, etc. for just this one multiplication.

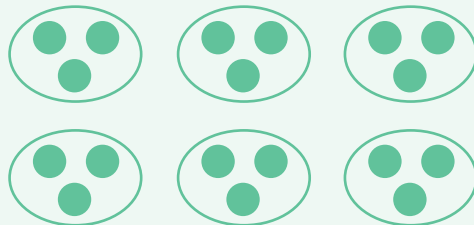


Six columns of three

? Can you think of another way to show three multiplied by six?



Three multiplied by six is $3 + 3 + 3 + 3 + 3 + 3$

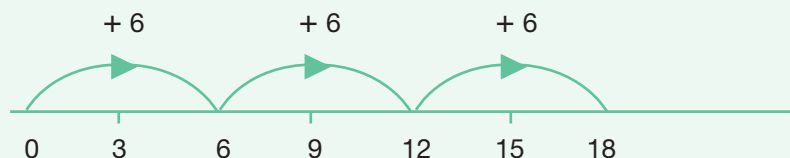


Six lots of three

It is likely that the question of changing around the pictures to six multiplied by three will arise, so, if you have time, support the child to shift around the array and regroup the eighteen counters into three groups of six. Ask the child to talk about how the picture has changed.

? How would you show three groups of six on a number line?

If the child doesn't know, show them how to take three hops of six.



$$6 + 6 + 6 = 18$$

Stress that this is six multiplied by three, but before we were finding three multiplied by six.

? If I asked you a question like 'What is nine threes?' and you didn't know that answer immediately, what could you do to work it out?

If the response indicates that the child has few mental strategies, and underdeveloped mental images for multiplication, you might want to repeat these activities in your next session, maybe choosing another number to multiply three by. You might want to build on what the child says about their favourite equipment to show multiplication.

Continuation for Years 5 and 6

You can show how the six times table can be found by doubling the three times table. The endings for the three times table are a bit awkward; every other one is an odd number. The answers to the six times table are all even numbers.

The nine times table can be made by adding answers from the three and six times tables.

$$2 \times 3 = 6 \quad 2 \times 6 = 12 \quad 2 \times 9 = 6 + 12$$

The endings of the nine times table are memorable:

9, 18, 27, 36, 45, 54, ...

? Can you see a pattern?

You can repeat the activities above (showing a whole range of pictures and images of multiplication) with facts like seven multiplied by eight that the child finds hard.



? Can you find even more ways of saying or drawing three multiplied by six?

? Can you explain those endings to the nine times table? Three multiplied by nine is twenty-seven and four multiplied by nine is thirty-six. Explain why the tens number goes up by one and the units number goes down by one.

Spotlight 4

Is not confident in recalling multiplication facts

Opportunity for: solving problems in a real-life context

At the shop

Time 10–20 minutes

Resources

- *Multiplication grid 1* (Resource sheet 35) or for Year 6 work use *Multiplication grid 2* (Resource sheet 36)
- L-shaped card
- *Things to buy* (Resource sheet 37)
- Different coloured pens, pencils, paper
- Coins (various denominations)
- Kitchen timer or stop clock

Key vocabulary

groups of	array
lots of	costs
multiplied by	how much

Teaching activity

‘Today we are going to do some shopping to help you to practise your multiplication tables.’

Show the child the money and check that they can recognise the coins.

? Do you do any shopping on your own? What do you buy?

Fill in the amounts of money on *Things to buy* (Resource sheet 37) to practise the tables you want. For example, to practise the two times table, make the lollies two pence each, for the ten times table make the cookies ten pence each, and so on.

? How much would ten lollies cost if they are two pence each?

If the child can’t work this out relatively quickly, show them how to use their L-shaped card to find two multiplied by ten on *Multiplication grid 1* (Resource sheet 35). For Year 6 work use *Multiplication grid 2* (Resource sheet 36).

? If you had twenty pence/fifty pence/one hundred pence and you wanted to spend all of it buying lots of just one item, what could you buy? (For example, ten balloons at ten pence each.)

Ask a range of questions to suit the child.

? How much would it cost to buy six packets of stickers?

? What could you buy with eighteen pence?

? If you bought two boxes of pencils at twenty pence a box, how much change would you get from sixty pence?

? Without using your L-shaped card, can you tell me how much ten balloons at eight pence each would cost?

Now ask the child to look at the things to buy and make up a multiplication question. Help the child to make up these questions using *Things to buy* (Resource sheet 37) for ideas.

If the child still has many facts to learn by heart, you could use a blank copy of *Multiplication grid 1* (Resource sheet 35) to practise tables again. Set the clock and explain that the child has to fill in the answers to the tables as quickly as they can. Then the next day they should try again and try to beat their time.

More-confident children could use *Multiplication grid 2* (Resource sheet 36). If you white out the numbers along the top, you could put the tables in a different order to make the task more challenging.

Spotlight 5: a learning check

Is not confident in recalling multiplication facts

Opportunity for: talking about multiplication and explaining images

Red race

Time 10–20 minutes

Resources

- At least one other child
- *Multiplication grid 1* (Resource sheet 35) or for Year 6 work use *Multiplication grid 2* (Resource sheet 36)
- Number cards 0–10 (Resource sheet 1)
- L-shaped card
- Different coloured pens, pencils, paper
- Cubes
- 2-cm squared paper
- Red dice, spinners

Check: does the child use key vocabulary?

- | | |
|---------------|--------------|
| groups of | equal groups |
| lots of | sets of |
| multiplied by | rows of |
| array | columns |
| divided by | |

Teaching activity

‘Today we are going to play a multiplication game, **Red race**, so that you get better at knowing your multiplication tables by heart.’

Number a red dice or red spinner 1 to 6 (or whichever multiples you want the children to practise). Each player will need an array cut out of 2-cm squared paper to keep their own score. Three rows of four is a good size for this purpose.

(If you are playing a game with a red dice with numbers 5 to 10, you need to make the paper array much bigger, perhaps ten rows of six.)

- The red dice will decide the size of the column for the array in each round of the game.
- Number cards 0 to 10 (or those to suit the children) are the multiplying cards and will decide the number of columns in each round of the game.

How to play

1. Players throw the red dice and draw a card to define the size of their array. For example:



Size of columns

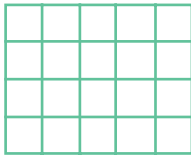


Number of columns
(the number to multiply by)



4 multiplied by 5

2. When every player has their two numbers, they write them down, for example, four multiplied by five or 4×5 . If any players have the same numbers, one of the players can throw again.
3. Each player makes an array with cubes, the right size for their dice and card numbers.
4. When all of the arrays have been made, the players take it in turns to tell everyone about their array. Everyone must listen very carefully.



I've made an array with five columns of four.

Four multiplied by five is twenty.

$4 + 4 + 4 + 4 + 4$

That is five hops of four along a number line.

If everyone agrees that the player said everything correctly, that player wins all of the cubes in one of their columns and places the cubes on their paper array.

If a player doesn't describe their array correctly, but shows some understanding, give them an appropriate number of cubes to acknowledge their efforts.

5. The first player to cover all of the squares on their paper array wins, but only if they can describe their paper array correctly! Ask the children to listen carefully.

If a child is not coping with the game, play alongside them, or play the game in cooperating pairs.

Note which numbers the child seems not to remember and give them some further experiences to practise those numbers, using any of the activities above.

Frequent repetition of numbers can aid the memory, so in one week you might ask the child just four calculations in odd moments throughout the week (and ask for help at home as well). For example, you could keep asking what is:

- three multiplied by seven?
- four multiplied by six?
- four multiplied by seven?
- three multiplied by nine?

Repeat these in any order until the child seems to remember them – but you will need to come back to these numbers in the following weeks as well.

Variations

- Instead of making the array with cubes, each player draws their array and has to make at least one other correct drawing using a different image, for example, hops along a number line or spots in groups.
- Instead of winning just one column, players keep a running total of all their numbers, so they score twenty with four multiplied by five. The player with the largest total wins. (A variation on this is that they only win if they can describe an array that would use almost all of their score, so a score of 109 might make an array of ten multiplied by ten with nine left over.)

? Can you explain to me how you would work out three multiplied by seven if you didn't have your grid and you didn't have any cubes?

? What is your favourite picture in your head for multiplication?

Learning outcomes

By the end of this set of activities, children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- recall multiplication facts;
- use a range of strategies to work out multiplication facts;
- use their mental images and language for multiplication confidently;
- apply multiplication facts in real-life situations.

Is muddled about the correspondence between multiplication and division facts, recording, for example: $3 \times 5 = 15$, so $5 \div 15 = 3$

Opportunity for: recognising relationships

Resources

- Board with space for four number sentences
- Four sets of number and symbol cards (Resource sheets 1, 2 and 8)
- Cubes or counters
- Number lines



Key vocabulary

multiplied by	steps of
divided by	change around
array	undoes
groups of	rows
hops of	columns

Teaching activity

Time 10–15 minutes

Explain to the child that today's activity will help them to find out more about the ways in which multiplication and division are linked.

- ? Can you tell me what you think it means if I say that multiplication and division are linked? Choose from the things on the table to show me what you mean.**

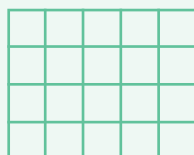
Follow on from what the child says, noting their idea of the link between multiplication and division.

Try to establish that one undoes the other. For example, if you multiply four by five you get twenty and if you divide twenty by five you get four.

- ? Can you draw something to show four multiplied by five that also shows twenty divided by five?**

Children might draw a number line or groups or an array. Follow on from what they draw to talk about the different calculations which their drawing shows.

If the child is confused, help them to lay out an array of twenty cubes or counters arranged as four multiplied by five: five columns of four cubes.



Four multiplied by five
 $4 \times 5 = 20$

- ? Can you show this same multiplication on a number line?**

If the child is unsure, draw a line for them.

? Can you tell me how big we need to make the hops along the line to show four multiplied by five?

Clarify that it is the four that is the number that is being multiplied, so the steps are four and there are five of them.



Help the child to draw the hops of four.

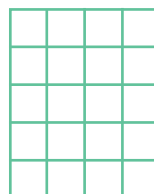
Clarify that this shows $4 + 4 + 4 + 4 + 4$ or 4×5 .

? Which other number sentences can we make with $4 \times 5 = 20$?

Talk the child through how you can look at the array the other way around and make five multiplied by four. (You can just rotate the array a quarter turn or move yourself to see it the other way around.)

‘Five multiplied by four looks like this.

We are looking at it the other way around.’



? Can you write a division number sentence that shows $4 \times 5 = 20$ or $5 \times 4 = 20$?

If the child is confused or records this incorrectly, use the array or the number line, or both if you have time, to model the divisions.



‘We can start from 20 and hop back in hops of four. We will need five hops of four to get to zero.’

$$20 - 4 - 4 - 4 - 4 - 4 = 0$$

‘So there are five groups of four in twenty. We needed to take five hops of four.’

? What if we do it the other way and make hops of five? How many hops will we need?

‘Let’s record all the number sentences.’

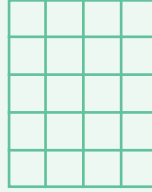
Put the four number sentences on the board, demonstrating how you use just three different number cards and you need four cards of each number, two $\times$ cards and two $\div$ cards and four = cards.

$4 \times 5 = 20$	$5 \times 4 = 20$
$20 \div 5 = 4$	$20 \div 4 = 5$

(You might find it helpful to take the board back to class to remind the child in odd moments about what it shows.)

If the child seems confused, ask them to look at the columns in the array.

Explain that we can divide this array into fives. Ask the child to move one column of five at a time.



? **How many fives in twenty?** ($20 \div 5 = 4$. *There are four groups of five in twenty.*)

? **Can you show me those four groups of five in the array?**

? **What have you learned today?**

? **What did you find hard today?**

? **Are there any words we used today that you are unsure about?**

Spotlight 1

Is muddled about the correspondence between multiplication and division facts, recording, for example: $3 \times 5 = 15$, so $5 \div 15 = 3$

Opportunity for: communicating with words and symbols

Three-card magic

Time 10–20 minutes

(Note: This Spotlight includes an optional activity about sharing apples, to use if the child needs further reinforcement. Additional time would be required.)

Resources

- Four sets of number and symbol cards
(Resource sheets 1, 2 and 8)
- Cubes or counters
- Board with space for four number sentences

Key vocabulary

multiplied by	change around
divided by	undoes
array	rows
hops of	columns
steps of	

Teaching activity

‘Today we are going to think about the two multiplication number sentences and the two division sentences that we can make with fifteen cubes.’

? How could you put out these fifteen cubes to make an array?

The array could be three rows of five cubes or five rows of three cubes. (It could also be one row of fifteen or fifteen rows of one.)

? Which two multiplication sentences can we make with this pattern?

? Which number cards do you need to make your sentences?

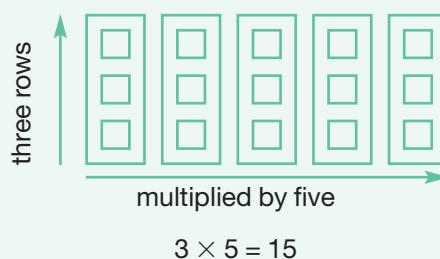
Help the child to record with cards:

$$3 \times 5 = 15$$

$$5 \times 3 = 15$$

Show how the numbers arise from the array.

If the child needs support in knowing which way around the array needs to be for these two multiplications, you can show the rows and columns like this:



? Which is the division card?

? Which division number sentences can we make?

? How can you make these cubes show a division sentence?

If the child needs help with this, show how the array can break down into fifteen divided into three groups of five ($5 + 5 + 5$) or fifteen divided into five groups of three ($3 + 3 + 3 + 3 + 3$).

Help the child to record and read:

$15 \div 5 = 3$ 'Fifteen divided by five is three groups of five.'

and

$15 \div 3 = 5$ 'Fifteen divided by three is five groups of three.'

grouping
model

? What if I move your cards around to make this number sentence? $3 \div 15 = 5$

? Can you read that number sentence to me?

? If we put the five after the equals would the number sentence be correct? Why not?

Establish that this sentence is three divided between fifteen.

This might be three pens divided between fifteen children!

That would not even be enough for one pen each, let alone five!

sharing
model

Sharing apples

If the child finds this challenging, try another real-life context.

? If we had fifteen apples to divide between three of us, how many would we have each?

Establish that you would have five each.

? If we had three apples and we had to divide them between fifteen people, would everyone get at least one whole apple, or would we have to cut them up?

Establish that each person would only have a small piece of apple.

'So if we write $3 \div 15 = 5$

we end up with an answer of less than one, and that doesn't fit with our cubes.'

Focus on the cubes again.

'We start with fifteen cubes. Let's put that first.'

$$15 \div 3 = 5$$

Help the child to talk you through the words of this number sentence.

? How many threes in fifteen, or fifteen divided by three, or how many groups of three are there in fifteen?

(Keep away from the phrase 'goes into' because this can cause confusion.)

If there is time, repeat the activity by looking at what happens if you try to divide five apples between fifteen people.

sharing
model

sharing
model

? What did you learn today using these number cards?

Establish that you had to keep using the same three numbers to make the three-card magic work.



? If we are doing some three-card magic and one of the numbers is twelve, what could the other numbers be? (Could be $3 \times 4 = 12$, or $12 \times 2 = 24$, or $12 \times 100 = 1200$, or...)

Spotlight 2

Is muddled about the correspondence between multiplication and division facts, recording, for example: $3 \times 5 = 15$, so $5 \div 15 = 3$

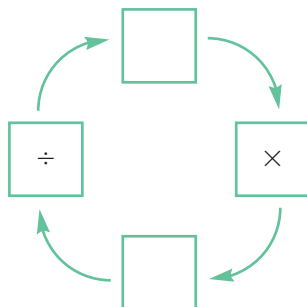
Opportunity for: recognising relationships

Round and rounds

Time 10–20 minutes

Resources

- 'Round and round' chart
- Number and symbol cards
(Resource sheets 1, 2 and 8)



Key vocabulary

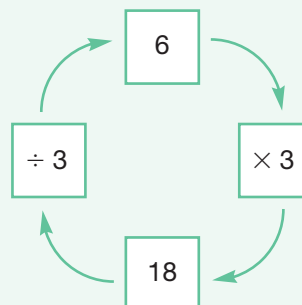
multiplied by	change around
divided by	undoes
array	rows
hops of	columns
steps of	reverse the numbers

Teaching activity

'Today we are going to do a bit more of the three-card magic that we did last time, and we are going to think about how division undoes multiplication.'

? Remind me what we found out before about three number cards going together in a special way.

If the child can't remember, remind them and see if they can find three cards which go together that you can use, for example 3, 6 and 18 (or another multiplication fact that the child finds hard to remember).



Demonstrate how to move around the chart saying the calculation, for example 'six multiplied by three is eighteen and eighteen divided by three gets you back to six'.

If the child isn't following this, talk it through once more, then ask the child to choose some other numbers for a multiplication fact they know, for example $2 \times 5 = 10$.

Help the child to 'read' the round and round chart and put in other numbers that go together.

? Do you think it will work if we go the other way round?

? Can you write twenty divided by five as a calculation?

If the child needs support for this, help them to write the calculation and do a few more examples, such as thirty divided by three and so on, each time starting with the 'round and round' chart.

- What is the correct way to write fourteen divided by two? Is it $14 \div 2$ or $2 \div 14$? How do you know?
- ? Can you read this: ? If it was about biscuits to divide between children, what might two divided by fourteen mean?

Use number cards to show that you can swap around multiplication calculations but you can't do that with division calculations.

$4 \times 5 = 20$ and $5 \times 4 = 20$

$20 \div 4$ is very different from $4 \div 20$. Stress that these two calculations have a different answer so the child must be careful not to swap the numbers around in divisions and expect the same answer.

Choose three more numbers and ask the child to put them on a 'round and round' chart and then write the two divisions that can be made from those numbers.

- ? Can you tell me what you learned today about some calculations where you can reverse the numbers and some where you can't?

Support the child to give examples of where reversing the numbers will not work.

Spotlight 3

Is muddled about the correspondence between multiplication and division facts, recording, for example: $3 \times 5 = 15$, so $5 \div 15 = 3$

Opportunity for: developing mental images

.....

Make a picture
Time 10 minutes

Resources	Key vocabulary	
● Board with space for four number sentences	multiplied by	steps of
● Number and symbol cards (Resource sheets 1, 2 and 8)	divided by	change around
● Cubes	divided into	undoes
● Number lines	array	rows
● Bead string	hops of	columns

Teaching activity

'Today we are going to think about some pictures that we can make to show divisions.'

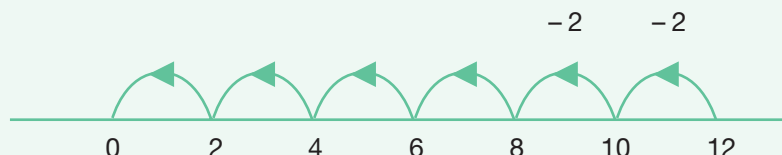
- ? What picture do you have in your head for twelve divided by two?
- ? Can you write twelve divided by two?
- ? How would you work out twelve divided by two?

Encourage the child to tell you about their own mental images. If we suggest using cubes to help, we might be pushing a child into a method of working that isn't closely related to what they do in their head.

? What is your favourite way of working out divisions?

If the child clearly needs more guidance, ask them to choose any of the equipment on the table, or suggest that they draw some kind of picture to show how they work it out.

If they are still finding this difficult, suggest that they draw a number line.



'Twelve divided by two can be done by stepping back in twos on the number line from 12.'

? Can you write in the numbers we land on when we step back in twos?

? How many steps of two did we need to take to get to zero?

'We needed six steps of two to get to zero, so twelve divided by two is six. Or we could ask, how many twos in twelve? Or we could ask, how many twos make twelve?'

? How many twos make twelve?

? So if we had twelve apples and we wanted to give two each to some children, how many children could have two each?

Make the calculation with number cards and then record it with a pen. Keep it to refer to and keep some of the pictures.

$$12 \div 2 = 6$$

? What happens if we write the division the other way around ($2 \div 12$)?

? Can you read that calculation? What picture does that make in your head?

? If we thought of it as apples, what would two apples divided into twelve look like?

Make sure the child understands that this calculation gives a completely different answer. The two apples would be cut up into small pieces and the answer is definitely not six! It is less than one, so it is a fraction (about 0.167).

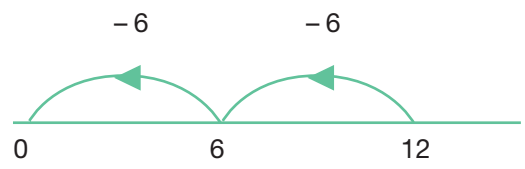
Working with the cards again, ask the child to rearrange the cards so that the calculation is correct and they are using use only those three numbers.

12	÷	6	=	2
----	---	---	---	---

? Can you read this division to me?

? What pictures do you have in your head?

You could either turn any array they have made a quarter turn, or suggest they draw a different number line with different steps.



? What is special about these two number sentences?
(They use the same three numbers. They go together.)

12	÷	2	=	6
----	---	---	---	---

12	÷	6	=	2
----	---	---	---	---

? Does this division fit with the others? Tell me why it does or why it doesn't.

2	÷	12
---	---	----

? Show me what you have learned today.

'If I tell you that four multiplied by six is twenty-four, can you write me two divisions that use just these three numbers?' Lay out the calculation with number cards.

Spotlight 4

Is muddled about the correspondence between multiplication and division facts, recording, for example: $3 \times 5 = 15$, so $5 \div 15 = 3$

Opportunity for: making connections

Multiplication square twos

Time 10 minutes

Resources

- Board with space for four number sentences
- Number and symbol cards (Resource sheets 1, 2 and 8)
- Cubes
- Number lines
- Sweets or counters
- Bead string
- Completed multiplication grid, for example, *Multiplication grid 1* (Resource sheet 35) or for Year 6 work use *Multiplication grid 2* (Resource sheet 36)

Key vocabulary

- | | |
|---------------|---------------|
| multiplied by | change around |
| divided by | undoes |
| array | rows |
| hops of | columns |
| steps of | |

Teaching activity

You need some number cards of multiplication facts which the child needs to practise, for example 24 (to go with 8 and 3), 18 (to go with 3 and 6), 21 (to go with 3 and 7).

Lay the number cards out on the table, 21, 24, 18 and so on, disregarding the lower numbers.
 'Today we are going to make some division calculations using these cards.'

'I want you to choose a number and show your division with that number using any of the equipment on the table, or you can draw a picture. We are also going to use these sweets – and I don't want to have to cut up any of the sweets! They are much too hard to cut so you must think carefully about the number you start with.'

? Which number will you choose to start with?

If the child needs help, suggest they start with twenty-one.

Help the child to count out twenty-one sweets and to put them in an array.



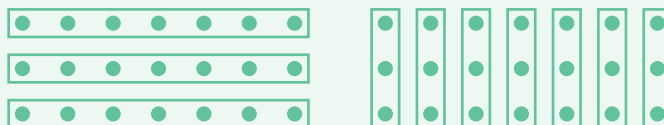
? Can you tell me something about our array of sweets?

Encourage the child to begin with the fact that there are twenty-one sweets.

Support the child to talk about seven columns of three sweets, that is $3 + 3 + 3 + 3 + 3 + 3 + 3$ or 3 multiplied by 7 or, if you look at it the other way around, $7 + 7 + 7$ or 7 multiplied by 3.

? Can you show me how we could use our array of twenty-one sweets to show a division?

Help the child to divide the array into three rows of seven and into seven columns of three.



? Which of these lower numbers could you choose to make a division calculation with twenty-one, making sure that we don't have to cut up the sweets?

The child should select the 3 card and the 7 card.

If the child needs support, you can ask them to look at their multiplication grid.

? Can you find 21 on the multiplication grid? Which two other numbers does it go with?

Work towards placing cards to give:

$$\boxed{21} \div \boxed{7} = \boxed{3} \text{ and } \boxed{21} \div \boxed{3} = \boxed{7}$$

? Why did you start that division with twenty-one?

Would it work if we put $3 \div 21 = 7$?

? Can you read this number sentence to me?

How could we have three sweets divided by twenty-one?

(Or three divided by seven? Or seven divided by twenty-one?)

Then you could record the two divisions on the board with space for the four number sentences.

Move on to the other cards (24 and 18 or whatever you have chosen for the child to practise).

Spotlight 5: a learning check

Is muddled about the correspondence between multiplication and division facts, recording, for example:
 $3 \times 5 = 15$, so $5 \div 15 = 3$

Opportunity for: discussing and explaining

Thinking threes

Time 5–15 minutes

Resources

- Sets of number cards that go together, i.e. two numbers and their product, for example 3, 4 and 12 (Resource sheets 1, 2 and 3)
- At least one other child
- Cubes
- Number lines

Check: does the child use key vocabulary?

multiplied by	steps of
divided by	change around
go together	undoes
array	rows
hops of	columns

Teaching activity

‘We are going to play a game called **Thinking threes**, and you will need to think very carefully to work out which three cards go together to make a trick.’

Lay the cards face up on the table (with the larger number cards together to make the choice of cards easier). Choose cards to suit the child. For a 5-minute game, you just need about eight tricks and a few odd cards.

Possible cards which you might use are:

2, 4, and 8	4, 4 and 16	5, 7 and 35
3, 4 and 12	4, 7 and 28	4, 8 and 32
3, 7 and 21	3, 9 and 27	

Include a few extra cards, such as a couple of 1s, a 10, another 12 and a 14.

How to play

1. Players take turns to choose three cards that go together, for example 10, 2 and 5.
2. Players explain why they chose those cards. For example, ‘I chose those cards because ten divided by five is two.’ (You can add that they must show their three numbers together on a number line or with an array or on a multiplication square.)
3. If everyone agrees that the three cards go together, the player wins that trick. If a player thinks the cards don’t go together, they must say why. They can use any equipment to show why the cards don’t go together and, if they are right, the player who tried to make the trick misses that go and the cards are put back on the table.
4. Play goes on like this until there are only a few cards left.
5. The winner is the player with the most tricks.

Variations

- Play with the larger-number cards in a bag. These cards are picked at random and the player must find the other two cards to make the trick.
- Play with a dice or spinner that makes one of the numbers, for example, a dice marked 2, 2, 3, 4, 5, 10.
- Play with one complete set of number cards 0 to 30.
- Play as a race. All the cards are put on the table, someone says 'Ready, steady, go' and the children race to make as many tricks as they can. (Don't play this with your best cards! Paper copies are fine.)

Scoring

Score 10 for every trick that is right.

Score negative 10 for any tricks that are wrong.

? Can you explain to me how you knew those three numbers go together?

Learning outcomes

By the end of this set of activities, children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- make two correct multiplication and two correct division number sentences with the same three numbers;
- understand that multiplication calculations can be done the other way around, but division can't (that is, multiplication is commutative but division is not);
- explain or draw their mental images of multiplication and division and understand the connections between these images;
- begin to understand that a division such as $2 \div 12$ would result in a number smaller than one.

Describes the operation of multiplying by ten as 'adding a nought'

Opportunity for: investigating numbers

Resources

- Interactive Teaching Program (ITP) – *Number Grid*, or paper copy of 100-square
- Coloured pens
- *Place value chart* (Resource sheet 39)
- Place value (arrow) cards
- Number cards 1–100 (Resource sheets 1, 2, 3, 4, 5, 6 and 7)

Key vocabulary

how many times?	group
multiply by ten	digit
lots of	row
move one place to the left	

Teaching activity

Time 20 minutes

Explain to the child that they are going to find out more about what happens to the digits of a number when multiplying by ten.

Using the ITP *Number Grid*, display a 100-grid.

With the child, count out loud in ones – highlighting the numbers as you count – until you have counted 'ten lots of one' (i.e. ten).

Repeat this activity with two, three and four (i.e. twenty, thirty, forty).

? Can you predict which number will be the answer to ten lots of five, six, seven, eight, nine?

If the child answers correctly, move on.

If the child doesn't understand or answers incorrectly, repeat the highlighting activity with some of these numbers to check the answers and reinforce understanding.

Have pre-prepared numbers 1–9 and 10–90 set out as indicated below, lining up the units, or use the place value chart on Resource sheet 39.

Look together at those pairs of numbers already explored, for example, two and twenty, four and forty.

Remind the child that the numbers in the second row are ten times bigger than those in the first row.

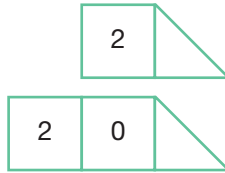
1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90

? What has happened to the '2' digit on this row (pointing to the tens row)?

If the child answers correctly, move on.

If the child answers incorrectly, repeat with other examples to ensure that they see the pattern and know that the digits always shift one place to the left when a number is multiplied by ten.

Now model with place value cards to reinforce understanding of the rule that when we multiply by ten all the digits move one place to the left. For example, select 2 and 20:



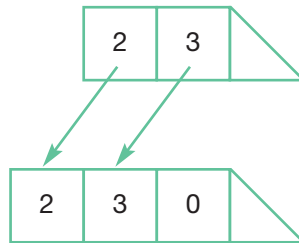
Position the twenty below the two, lining up the arrow heads.

Ask the child to choose a different single-digit number and the number which is ten times bigger, setting them out as shown above.

? Now that you know a way of multiplying any number by ten, can you tell me the answer to twenty multiplied by ten?

Ask a range of questions, for example: seventy times ten; nine lots of ten; the answer to thirty multiplied by ten, and so on.

Select a tens and a units place value card, for example 20 and 3. Combine them to make the number 23 and then multiply the number by ten. Emphasise that the digits move one place to the left.



Ask the child to make further two-digit numbers and multiply each by ten, for example 34×10 , 86×10 .

At each stage, ask the child to explain what happens to the digits when the number is multiplied by ten.

If the child understands, move on to three-digit numbers.

Repeat the process, asking the child to make a three-digit number and then multiply it by ten, for example 123×10 , 287×10 , 789×10 .

Further examples can be carried out at home.

? What did you enjoy today?

? What is very important for us to remember for next time?

Spotlight 1

Describes the operation of multiplying ten as ‘adding a nought’

Opportunity for: recognising relationships

.....

Non-musical chairs

Time 10–15 minutes

Resources

- Three chairs
- A4-size number cards 1–9
- Two A4-size 0 cards
- Place value (arrow) cards

Key vocabulary

- | | |
|----------------------------|---------|
| how many times? | lots of |
| multiplied by ten | group |
| move one place to the left | digit |

Teaching activity

‘We’re going to do an activity with these chairs about multiplying by ten. One chair will be the “units chair” and another will be the “tens chair”.’

Ask the child to choose a card from the 1–9 number cards, for example 4.

Then ask them to sit on the ‘units chair’, displaying their card.

? What happens if we multiply that number by ten?

Reinforce the knowledge that the digits shift one place to the left when we multiply any number by ten.

If the child is unable to answer correctly, use place value cards to reinforce the process.

Ask the child to demonstrate their answer (40) by selecting a 0 card and placing it on the units chair, and then sitting in the correct place, that is, on the tens chair.

? What happens if we multiply the new number by ten?

Again reinforce the knowledge that the digits shift one place to the left when we multiply any number by ten.

? What did you learn today?



Play the game with the whole class, going on multiplying by ten up to a million.

Spotlight 2

Describes the operation of multiplying ten as ‘adding a nought’

Opportunity for: making mathematical connections

.....

Chairs again

Time 10–15 minutes

Resources

- Three chairs
- A4-size number cards 1–9
- Two A4-size 0 cards
- Place value (arrow) cards

Key vocabulary

- | | |
|----------------------------|-------|
| how many times? | group |
| multiply by ten | digit |
| lots of | row |
| move one place to the left | |

Teaching activity

‘Today we’re going to do that activity with the chairs again, so that we can learn more about multiplying by ten. But today we are going to work with two-digit numbers so we will need a “hundreds chair”.’

? Can you remember what a two-digit number is?

Ask the child to choose two cards from the 1–9 number cards, for example 4 and 2.

Then ask them to put one of the cards on the ‘units chair’ and the other on the ‘tens chair’, for example 4 on the ‘tens chair’ and 2 on the ‘units chair’.

? What two-digit number have you made?

? What happens if we multiply that number by ten?

Reinforce the knowledge that the digits shift one place to the left when we multiply any number by ten.

If the child is unable to answer correctly, use place value cards to illustrate the process.

Ask the child to demonstrate their answer (420) by selecting a 0 card and placing it on the ‘units chair’, and moving the 2 to the ‘tens chair’ and the 4 to the ‘hundreds chair’.

? What happens if we multiply another two-digit number, for example seventy-four, by ten?

Reinforce the knowledge that the digits shift one place to the left when we multiply any number by ten.

? What was the same about multiplying two-digit numbers by ten and what we did last time multiplying one-digit numbers by ten?

Spotlight 3

Describes the operation of multiplying ten as 'adding a nought'

Opportunity for: reasoning; deciding whether answers are reasonable

Bingo

Time 10–20 minutes

Resources

- *Bingo! game sheet* (Resource sheet 38)
- Place value (arrow) cards (optional)

Key vocabulary

multiplied by ten
digit
move one place to the left

Teaching activity

'We're going to play a **Bingo** game today, so that you will learn more about how the digits move when you multiply by ten.'

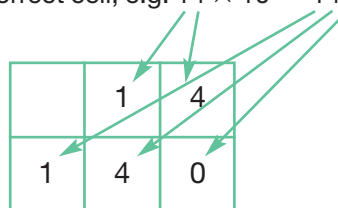
The grids of numbers and a set of recording grids can be found on *Bingo! game sheet* (Resource sheet 38).

First demonstrate the activity, explaining that numbers in the right-hand grid are numbers from the left-hand grid multiplied by ten, but muddled up!

The child should then work either individually or with you.

How to play

- 1 Select a number from the left-hand grid on the *Bingo! game sheet* and record it in the first row of the recording grid. Ensure that each digit of the number is entered in the correct cell.
- 2 The player then multiplies their number by ten and checks their answer by finding the match in the right-hand grid. The answer is then recorded in the second row of the recording grid, ensuring that each digit of the number is entered in the correct cell, e.g. $14 \times 10 = 140$:



If the child has difficulty, supplement the activity with further examples using place value cards.

- 3 Correct answers are shaded or crossed out on the right-hand grid. You could challenge the child to get from the left to the right, or make a pattern.

Reinforce the process of digits shifting one place to the left when a number is multiplied by ten.

? There is something very important that happens to the digits when we multiply by ten. Can you remember what it is?

Spotlight 4

Describes the operation of multiplying ten as 'adding a nought'

Opportunity for: recognising patterns

.....

On the calculator

Time 10–15 minutes

Resources

- Calculators

Key vocabulary

- | | |
|------------------|---------|
| multiply by ten | zero |
| move to the left | predict |

Teaching activity

'Today we are going to make some patterns on the calculator by multiplying by ten, a hundred and a thousand.'

? If you enter 4 on the calculator and multiply it by ten, what do you predict the answer will be?

Follow on from what the child says. Try several examples of multiplying by ten, talking about the zeros and the number moving to the left.

? What do you think will happen if we multiply the number by a hundred? What about a thousand?

Record with the child:

$$\begin{aligned} 9 \times 10 &= 90 \\ 9 \times 100 &= 900 \\ 9 \times 1000 &= 9000 \end{aligned}$$

Try several examples and, if you have time, you might want to explore with the child what happens if you key in:

- 9000 and divide by a thousand
- 900 and divide by a hundred
- 90 and divide by ten

If the child is struggling with this, give them much more time and opportunity to explore multiplying and dividing on a calculator, encouraging this to be a task to do at home.

? What did you enjoy today? Are you getting better at multiplying by ten?

Spotlight 5: a learning check

Describes the operation of multiplying ten as 'adding a nought'

Opportunity for: discussing and explaining

Move left

Time 15–20 minutes

Resources

- Two sets of number cards 1–9 (Resource sheet 1)
- Two sets of cards saying $\times 10$, $\times 100$ and $\times 1000$ (all the same size)
- Lots of 0 cards
- At least two children
- Large score charts (see below), big enough for the number cards to fit
- Timer
- Calculators to help with scoring

Check: does the child use key vocabulary?

- | | |
|----------------------------|-------|
| how many times? | group |
| multiplying by ten | digit |
| lots of | row |
| move one place to the left | |

Score chart					
Total:					

Teaching activity

'This is a game called **Move left**, and it will help you to get much better at multiplying by ten.'

The game can be played cooperatively in pairs, or children can see who gets the highest score, or you could set a timer so that the child or pair can try to beat their best time.

Place the number cards in one face-down pile and the multiplying cards in another face-down pile.

How to play

1. Each player or pair takes a number card from the face-down pile, for example 6. This is placed in the first row in the right-hand space on the score chart.
2. Then each player takes a multiplying card from the face-down pile, for example $\times 100$.
3. Each player or pair now multiplies their one-digit number by their multiplying card number (in this example, 6×100). They then move their 6 card to the left and put in two 0 cards in the empty columns.
4. Each player or pair can record their answer, 600 in this case, or just leave their cards on the score card.

5. Then each player takes two number cards and places these in the second row, making a two-digit number. Again, they take a multiplying card and move the two number cards left, putting in 0 cards.
6. Then each player takes three number cards to make a three-digit number and multiplies that by taking another multiplication card.
7. Then they add up their score (using a calculator if necessary).

Variations

- Play to see who can get the lowest score.
- Play the other way around. Call it 'Move right' and start with the number cards at the right and lots of zeros and have 'divide by' cards, such as $\div 10$, $\div 100$, $\div 1000$.

Learning objectives

By the end of this set of activities children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- describe the effect on the digits of multiplying a number by ten;
- understand why whole numbers multiplied by ten have a zero in the units position.

Does not apply partitioning and recombining when multiplying, for example 14×3 is calculated as $(10 \times 3) + 4 = 34$ or $14 \times 3 = 312$, confusing the value of two-digit numbers

Opportunity for: developing mental images

Resources

- Squared paper
- Scissors, ruler
- Cubes (some in 'ten trains')
- Place value (arrow) cards
- Glue stick

Key vocabulary

- | | |
|---------------|-----------------|
| multiplied by | row |
| altogether | column |
| array | equal groups of |

Teaching activity

Time 15–20 minutes

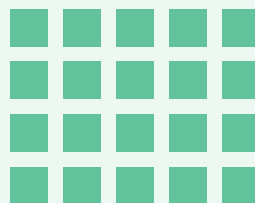
Note: It will be helpful right through this teaching sequence to have a note of the kinds of errors which the child has been making in class when doing grid multiplications. This information will be needed in Spotlights 4 and 5.)

Explain to the child that today's activity will help them to get better at multiplying numbers using a method which they have used before when they made arrays or grids, but this time using larger numbers.

'We are going to use lots of multiplying facts, including using multiplying by ten to make the multiplying easier.'

? Can you make an array using anything on the table to show four multiplied by five?

If the child finds this difficult, help them to make an array with cubes (ideally without joining the cubes together – just put them in rows in ones).



? Can you describe your array to me? (Clarify that it is four multiplied by five, so that is five equal groups of four or, if you look at it the other way around, it is four equal groups of five.)

? How many cubes are there altogether? How can you be sure you are right?

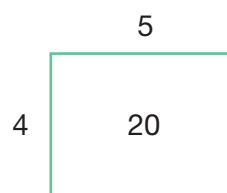
If the child starts to count in ones, stop them and ask them if they can find a quicker way, for example, counting in fours: four, eight, twelve, sixteen, twenty.

'There are twenty cubes altogether.'

? Can you draw this array of cubes on squared paper?

How many rows are there? (Four)

How many columns? (Five)



Record the four by five array on squared paper with a total of twenty squares and keep the cube and paper array for later.

? **Do you remember doing this when we worked on finding the area of shapes?** (Ideally remind the child about any specific activities they did.)

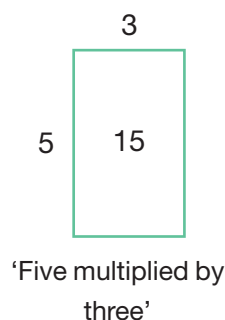
If the child seems unclear about finding the area of a rectangle, use squared paper to draw some rectangles, for example two by two and three by four. Talk the child through finding the area of these rectangles. Ask the child to write the area in the middle of each rectangle in square centimetres.

Emphasise that to find the area we must count every square.

Show how to do this for a 3×4 rectangle. Show how this is $3 + 3 + 3 + 3$.

‘Make a rectangle on squared paper four centimetres by five centimetres. We worked out that to find the area of a rectangle like this, we multiplied the length of one side by the length of the other side (4×5) because this is $4 + 4 + 4 + 4 + 4$.’

Build up a range of rectangles on centimetre-squared paper, writing the number of rows and columns, and putting the total number of squares in the middle of the rectangle, for example five multiplied by three, six multiplied by three, as well as the four by five rectangle above. These will be used later so you might want to stick them onto plain paper.

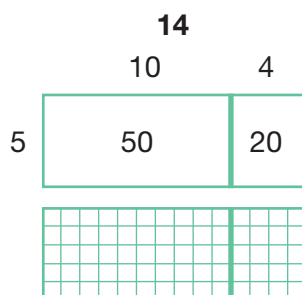


It is crucial that children are familiar with writing the number of rows and columns outside the array, and the total number of squares in the middle of the rectangle. If the child seems unsure about this, do some more examples using accessible numbers, such as 2×4 and 2×6 .

Using one of the examples that the child might manage to work with, make the number of columns ten more, for example 5×4 becomes 5×14 . Ask the child to make this array with cubes, not joined together, adding ten more columns to the 5×4 array (adding halved ‘ten trains’ is quickest), while you draw it on squared paper. You might want to put the ten rods to the left of the 5×4 .

Using a ruler, slightly separate the four original columns from the ten columns to show how the array of cubes can be split into 5×10 and 5×4 and record that on the paper array with a vertical line.

‘Here is 5×10 and here is 5×4 .’



Using place value cards, demonstrate how fourteen can be partitioned (or split) into ten and four.



Count the number of cubes, counting the columns, establishing that the child already knows $5 \times 4 = 20$.

‘Look carefully and tell me the number of rows and columns in the larger part of the array.’

Check that the child understands that this array is 5×10 , and that it is fifty cubes/squares.

? Where shall we record the fifty?

? Now what can we do to find out how many cubes there are altogether in our array?

Help the child to recombine the tens and then the ones with place value cards.

If the child cannot see that the fifty and twenty now need to be added, show the fifty plus twenty cubes laid out in the array and count them in fives with the child.

Repeat this with another of the arrays. For example, make the 5×3 array into 5×13 , again putting the new ten rods to the left of the original cubes.

Keep at least one of the larger arrays showing the partitioning into ten and a bit.

		13	
		10	3
5	50	15	

? Do you think we would get the same answer if we split the array in a different place?

? Can you sketch the array and we will partition it in a different way?

? Why do you think we split the larger arrays into ten and a bit?

Make sure that the child sees that the thirteen could be partitioned into nine and another four, or eight and five, but by splitting it at ten and a bit we make the multiplying easier, because multiplying by ten is easier than multiplying by eight or nine.

? What have you learned today? What would you like more practice with?



? Why do you think this way of multiplying works?

Spotlight 1

Does not apply partitioning and recombining when multiplying, for example 14×3 is calculated as $(10 \times 3) + 4 = 34$, or $14 \times 3 = 312$, confusing the value of two-digit numbers

Opportunity for: recognising relationships

.....

Split them up

Time 15–20 minutes

Resources

- Squared paper
- Ruler
- Cubes
- At least two sets of place value (arrow) cards
- 100-square
- *Place value chart* (Resource sheet 39)

Key vocabulary

partition	tens
split	ones
recombine	estimate
add together	

(Note: Keep a record of errors that the child makes during the Spotlights or classwork, so that you can reflect on them.)

Teaching activity

‘Today we are going to use the place value cards to partition numbers before we multiply them. This will help you when you want to multiply larger numbers.’

Put at least two 10 and two 20 cards on the table with some 1s.

? Can you use the place value cards to make some numbers larger than ten, but less than thirty?

Help the child to make numbers such as fourteen and twenty-three.



? Show me how you can partition those numbers into tens and ones.

If the child finds this difficult, use a place value chart to show how numbers are made up of tens and ones. Find the numbers on a 100-square as well. For example, demonstrate how 24 is made up of 20 and 4.

For the rest of the lesson, you could keep just to teen numbers if you think this will help. Help the child to record some of the partitioning.



Partitions (or splits) into ten and four

? Now can you recombine them again to make fourteen?

When the child seems confident with the place value cards, choose one of the numbers, for example fourteen.

? If you wanted to work out fourteen multiplied by three, which number would you partition to make the calculation easier? How can we partition fourteen?

Sketch a grid on paper, explaining that it doesn't need to be drawn to scale, it just needs to be big enough to write the numbers clearly. Encourage the child to talk through with you how fourteen can be partitioned into ten and four, then each part can be multiplied by three.

Put appropriate place value cards on the grid.

×	10		4	
3	3	0	1	2

? Can you show me what we can do next to recombine the numbers?

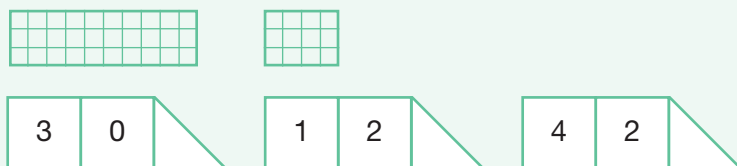
If the child is finding this hard, relate the grid and the place value cards to cubes laid out on the table.

'Three multiplied by ten is thirty and three multiplied by four is twelve.'

? About how big is your answer going to be? About forty? Or closer to a hundred?

? Which numbers do we have to add together to find out how many cubes there are altogether? (The thirty and the twelve to make forty-two.)

If the child is unsure about which numbers have to be added together, go back to the grid and the place value cards and relate these to the two groups of cubes, thirty cubes and twelve cubes.



? Can you show me how you would recombine the place value cards to find the total number of cubes?

? How did making an estimate of the answer help you?

If the child is unsure how to check, show how making an estimate of the calculation first can help them to see if their answer is roughly right. We can also check by doing a calculation a different way.

If you have time, repeat with other numbers, for example 13×5 .

Encourage the child to:

- estimate the answer first;
- sketch the grid – not to scale;
- check their answer. (Is it close to the estimate and can they check it by doing it another way?)

? When we are multiplying a number with two digits, what is it that we can do to make the multiplying easier? (Try to get at the idea of partitioning the larger number into tens and ones, and also that making an estimate first can help us to check our actual answer.)



? **What do you think would be the most helpful way to do the partitioning for multiplying twenty-seven by three?** *(For example, would it be easier to split twenty-seven into twenty and seven or nineteen and eight? Why?)*

? **Why is it helpful to partition numbers when we multiply?**



? **On my multiplication grid, one of the totals I've put on the grid after multiplying is twenty-four. What could the numbers be that I am partitioning to multiply?**

Spotlight 2

Does not apply partitioning and recombining when multiplying, for example 14×3 is calculated as $(10 \times 3) + 4 = 34$, or $14 \times 3 = 312$, confusing the value of two-digit numbers

Opportunity for: developing mental images

Number line multiplying

Time 10–20 minutes

Resources

- Squared paper
- Cubes
- Place value (arrow) cards
- *Place value chart* (Resource sheet 39)

Key vocabulary

partition	altogether
split	array
recombine	row
add together	column
tens	equal groups of
ones	estimate
multiplied by	

Teaching activity

‘We are going to do some more multiplying with two-digit numbers and today we are going to use a number line to help us.’

? Can you make a two-digit number with the place value cards?

? Can you partition your number into tens and ones?

If the child seems to be unsure, you can help them to partition a few more examples, but then move on.

? Can you make the number that has one ten and three ones? What is it?

? Can you write the calculation thirteen multiplied by three? (13×3)

? Make an estimate. Will the answer be about thirty or about seventy?

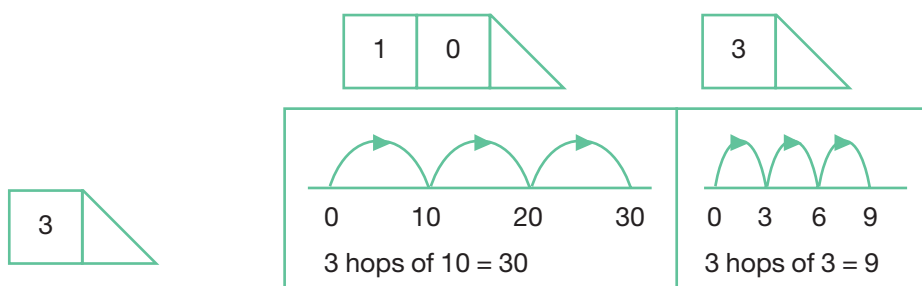
? Which number would you choose to partition to make the multiplying easier?

Help the child to partition thirteen into ten and three.



? Can you give me a rough estimate of what the answer is going to be? Somewhere around thirty or forty? Somewhere around a hundred? Which would be closer?

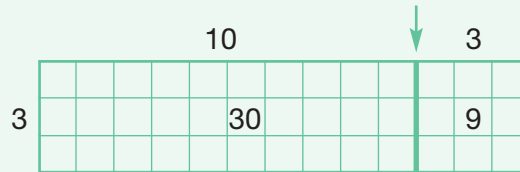
Draw a large grid, explaining that you are making it large enough to draw number lines in it. You can ask the child which numbers you need to write to the side and top of the grid, or help them to place their place value cards appropriately.



You or the child can place the number line hops.

- ? **Which place value cards do we need now to show the answers to the multiplication calculations we have done? (30 and 9)**

If the child still needs more support, you could ask them to make the array of 3×13 with cubes and show how that is partitioned.



- ? **Can you show me how you would recombine these place value cards?**

If the child cannot make the answer thirty-nine, you will need to do more examples of partitioning and recombining.

- ? **So what is the answer to thirteen multiplied by three?**



- ? **Was that quite near your estimate?**

- ? **Did it help you to do this calculation on a number line?**

- ? **If I draw a much smaller grid, can you multiply with the same numbers, 13×3 , without a number line?**

- ? **How will you partition the thirteen?**
Can you draw in the line
between the ten and the three?



Talk the child through the positioning of the numbers.

Talk about the sizes of the two rectangles, establishing that they are 3×10 and 3×3 .

- ? **Does it matter that our rectangles aren't exactly measured, they are just a sketch?**

Help the child to go through the grid multiplication to gain confidence in using a grid and positioning numbers appropriately.

- ? **Was our actual answer near our estimated answer?**

- ? **What helped you to learn something new today?**

Spotlight 3

Does not apply partitioning and recombining when multiplying, for example 14×3 is calculated as $(10 \times 3) + 4 = 34$, or $14 \times 3 = 312$, confusing the value of two-digit numbers

Opportunity for: communicating with words, images and symbols

Three times larger

Time 10–15 minutes

Resources

- Squared paper
- Ruler
- Base 10 equipment, or bundles of straws, bead strings
- Place value (arrow) cards

Key vocabulary

partition	multiplied by
split	altogether
recombine	array
add together	three times larger
tens	estimate
ones	

Teaching activity

‘Today we are going to do some multiplying using the grid method. We are going to use the Base 10 equipment to help us.’

‘Let’s start by making a two-digit number to multiply.’

? Can you make a two-digit number with the place value cards?

If the child finds this difficult, spend a few minutes making some two-digit numbers. Make sure they understand that the size of the ten rod is linked to its value of ten. Make sure they can put out equipment to the value of the place value cards.



Choose one of the numbers, for example eighteen, and use that to multiply by three.

? Can you make a sketch of the grid we need for multiplying eighteen by three?

? Can you make an estimate?

You might want to suggest that eighteen is almost twenty, so three lots of twenty would be $20 + 20 + 20 = 60$.

Therefore, if the actual answer isn’t near sixty, something has gone wrong.

Remind the child that the rectangle must be big enough to write the numbers in, and it is only a sketch so it doesn’t need to be to scale.

? Which number is it best to partition? Why? ($10 + 8$ is ideal.)

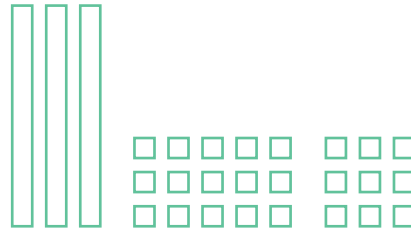
? Can you write the numbers that we are going to multiply around the grid?



? **Can you put out eighteen in the Base 10 equipment?**

Help the child to select one ten rod and eight unit cubes and place these on or by the grid.

'We are going to make this eighteen three times larger.'

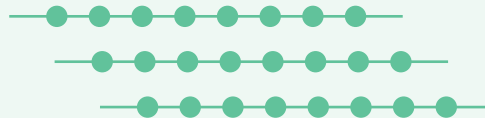


? **Can you make the ten rod three times bigger? What is ten multiplied by three?**

? **Why did you write thirty in that rectangle? (Clarify that this is three tens.)**

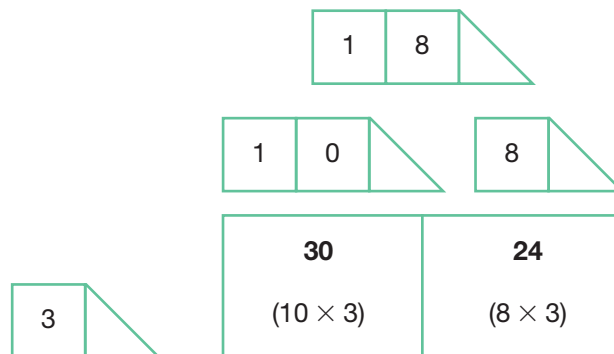
? **How can we make eight unit cubes three times bigger?**

If the child needs help with that, encourage them to count in eights three times, or make an array eight by three. Or use the model which the child tends to prefer for support, for example bead strings.



'Three lots of eight is twenty-four.'

If it will help, use place value cards to clarify:



? **Why did you write twenty-four in that rectangle?**

Help the child to use a wide range of the key vocabulary above.

? **Which numbers do we need to recombine to find the answer to eighteen multiplied by three?**

Support the child using place value cards if necessary, to recombine 30 and 24.

? **So what is the answer to eighteen multiplied by three? (54)**

? **Is that close to our estimate? So we are probably right!**

? **How could we record eighteen multiplied by three using just one line?**



Support the child to write:

Eighteen multiplied by three is fifty-four
 $18 \times 3 = 54$

If they understand the partitioning well, you could write:

' 18×3 can be written as (10×3) add (8×3) .'

This shows how the two separate multiplications came from the place value cards and/or the equipment.

(Note: Remember to keep a note of some of the errors which the child is making.)



- ? Do the multiplication twenty-seven multiplied by three using the grid method, but partition the numbers in three different ways. Will you always get the same answer?**
- ? When you get your answers from the multiplications in the grid, why do you add the numbers together?**

Spotlight 4

Does not apply partitioning and recombining when multiplying, for example 14×3 is calculated as $(10 \times 3) + 4 = 34$, or $14 \times 3 = 312$, confusing the value of two-digit numbers

Opportunity for: making decisions

Using brackets

Time 10–15 minutes

Resources

- Squared paper
- Cubes
- Place value (arrow) cards
- Base 10 equipment
- Bead strings
- Number cards (Resource sheets 1, 2 and 3)
- A note of errors which the child makes in class

Key vocabulary

partition	multiplied by
split	altogether
recombine	array
add together	row
tens	column
ones	equal groups of

Teaching activity

(Note: Remember you will need actual examples of errors which this child is making for this and/or the next session.)

Put out a few number cards, for example 16, 27, 24, 35, and some lower numbers to multiply by, maybe 3, 4 and 5.

‘I want you to choose some numbers that we can multiply together using the grid method.’

Help the child to select appropriate numbers, for example twenty-seven and four.

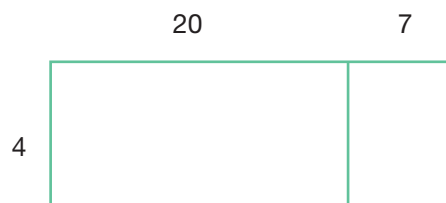
If the child is having trouble with recombining after multiplying, you might want to start with lower numbers, perhaps twenty-one multiplied by three.

? Which number shall we partition? Why?

If they need help with the partitioning, you might need to spend some time on that in another session. Do the partitioning and move on.



? Can you sketch a grid for twenty-seven multiplied by four?



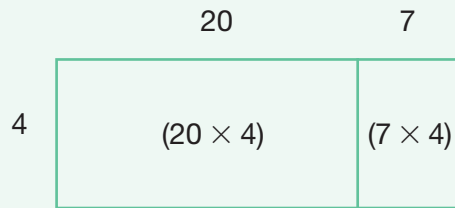
Show the child how to write the partitioning using brackets.

‘What we did was partition twenty-seven into twenty and seven and then multiply by four.’

‘We can write that like this:

$$27 \times 4 = (20 \times 4) + (7 \times 4)$$

If the child does not understand, demonstrate clearly, maybe marking those multiplications on the grid.

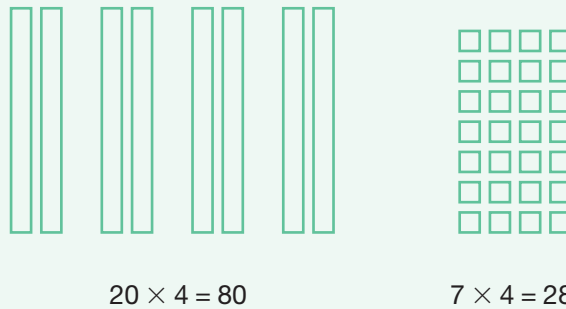


You might want to show these numbers with some kind of equipment that the child seems to prefer.

This is twenty-seven:



Twenty-seven made four times bigger is:



$$20 \times 4 = 80$$

$$7 \times 4 = 28$$

- ? What do we need to do now with the eighty and the twenty-eight?**
- ? Show me how you can recombine the eighty and the twenty-eight.**
- ? When we were partitioning the twenty-seven, why did we write it as twenty add seven?**

Support the child in talking about the values of the tens and the ones.

Demonstrate how the brackets help us to find the answer.

$$\begin{aligned}
 27 \times 4 \\
 &= (20 + 7) \times 4 \\
 &= (20 \times 4) + (7 \times 4) \\
 &= 80 + 28 \\
 &= 108
 \end{aligned}$$

Try some more calculations using brackets, for example:

$$\begin{aligned}
 17 \times 5 &= (10 + 7) \times 5 = \dots \\
 34 \times 6 &= (30 + 4) \times 6 = \dots
 \end{aligned}$$

(Note: The following text might relate to some of the errors that the child is making in their classwork. If that is the case, you might want to use this text. If the child isn't making these particular errors, you might want to move on to Spotlight 5, or look at specific errors which the child is making. It is important to address the errors that the child actually makes, not just suggestions of errors they *might* make!)

This text addresses one common place value error:

23 partitions into 2 and 3, so 23×4 is (2×4) added to (3×4) .

Help the child to work this out as far as $8 + 12$.

? Can you draw a grid for 23×4 ?



? What is the answer to 23×4 ? Is it $8 + 12$?

? Where did we go wrong with partitioning 23 into 2 and 3?

If you need to do another example showing place value errors, you could go back to 27×4 as above.

	20	7
4	(20×4)	(7×4)

? Why did we partition twenty-seven into twenty and seven? Why couldn't we partition twenty-seven into two and seven?

	2	7
4	$(2 \times 4) = 8$	$(7 \times 4) = 28$

So twenty-seven multiplied by four is eight add twenty-eight, which makes thirty-six.

Support the child to see that 36 is *not* a good estimate for 27×4 .

This is one way of making an estimate of the answer: 'twenty-seven is close to twenty-five, so twenty-five multiplied by four will be close to twenty-seven multiplied by four.'

$25 \times 4 = 100$ and an estimate for 27×4 must be close to 25×4 , so it must be close to 100.'

? Can you explain to me some of the things that we have to do very carefully when we are multiplying using this grid method?

? Look at this page in your maths book and find something you did wrong.

? Why do you think that was wrong?

? Is there anything you think you need more help with?



Spotlight 5: a learning check

Does not apply partitioning and recombining when multiplying, for example 14×3 is calculated as $(10 \times 3) + 4 = 34$, or $14 \times 3 = 312$, confusing the value of two-digit numbers

Opportunity for: communicating in symbols and words

Right or wrong?

Time 10–15 minutes

Resources

- Some prepared multiplications (relate these to errors which the child has made in class)
- Cubes
- Place value (arrow) cards

Check: does the child use key vocabulary?

partition	multiplied by
split	altogether
recombine	array
add together	row
tens	column
ones	equal groups of

Teaching activity

(Note: Remember to refer to errors that the child has been making in class.)

‘This grid method for doing multiplications is very useful, because it helps us to be able to multiply two-digit numbers, which would be difficult to do in our head. But we must be careful to do the grid method without making a mistake. I’m going to show you some multiplications done with a grid and some of them are right, but some are wrong.’

You could start with some correct grids but with some numbers missing.

14×6	$\times$	10	4
	6		(4×6)

34×4	$\times$	30	4
	4	$(30 \times)$	$(\times 4)$

? Can you tell me how you know you are right?

If the child needs help, let them choose another way to work the grid out, perhaps with number lines or arrays of cubes.

? Can you explain to me how you would draw a grid to work out thirty-seven multiplied by five?

? Can you talk me through how you would work it out?

? How can you check that you are right? What would be an approximate calculation? (Perhaps smaller than 40×5 but larger than 30×5 .)



You might want to focus on some of the specific errors which the child is making in class.

It is probably only relevant to work through the following text if the child you are working with is making this type of error.

'You didn't always partition the two-digit number correctly.'

'Is this right?'

18×3 is

1×3 added to 8×3

$3 + 24$

So 18×3 is 27.



Encourage the child to make an estimate in their head of 18×3 . 'So our answer needs to be almost 60, not 27.'

? Is this grid right or wrong for 15×4 ?

×	1	5
4	$(1 \times 4) = 4$	$5 \times 4 = 20$

So 15×4 is $4 + 20$ which is 24.



Encourage the child to estimate what four multiplied by fifteen might be.

'Let's make some grids with numbers missing (*and/or errors, whichever the child needs to focus on*) to take back to class, and we will ask other children if they can solve our missing number puzzles.'

Learning outcomes

By the end of this set of activities children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- draw an appropriate grid to multiply a two-digit number by a one-digit number;
- partition two-digit numbers appropriately and place these on the grid;
- explain which numbers are recombined;
- check a multiplication by making an estimate;
- identify errors and be aware of their own errors;
- start to use brackets to show how the numbers are partitioned and then multiplied, then recombined.

Assumes that the commutative law holds for division also, for example assuming that $15 \div 3 = 5$, so $3 \div 15 = 5$

Opportunity for: communicating mathematics in words and symbols

Resources

- Number cards and symbol cards
(Resource sheets 1, 2 and 8)
- Number lines
- Cubes

Key vocabulary

- other way around
- array
- multiplied by
- divided by
- divided between
- count back
- hops back

Teaching activity

Time 15–20 minutes

Explain to the child that the activity today will help them to learn more about ways in which we can rearrange numbers in a number sentence.

- ? Which number sentences can you remember where you can rearrange the numbers and you still get the same answer?**

If the child needs support with this, show a + card and lay out an addition, for example: $3 + 4 = 7$

- ? What if we reverse the three and the four? Will we still get the same answer?**

Clarify that four plus three is also seven.

Let the child suggest a few more examples, such as five plus two and two plus five.

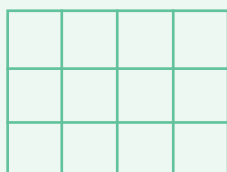
‘So we can add numbers in any order and we still get the same answer.’

- ? Can you make more than one multiplication number sentence with four, three and a × (‘multiplied by’) card?**

- ? Does three multiplied by four give the same answer as four multiplied by three?**

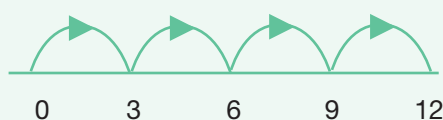
- ? Can you make something with these cubes or draw something to show me how you know you are right?**

Support the child to make an array and hops along a number line.



$$3 \times 4 = 4 \times 3$$

a hop of three, four times 3×4



a hop of four, three times 4×3



‘If you make three hops of four you land on 12, and if you make four hops of three you land on 12.’

- ? **Can you think of some number sentences that don't have the same answer when you put them round the other way?**

If the child has difficulties with this, support them with some examples of subtraction number sentences.

'Twelve subtract four equals eight, but if you change around the twelve and the four to four subtract twelve, what answer do you get?'

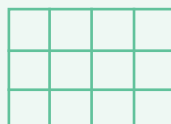
- ? **What division number sentences can you make with twelve and three?**

- ? **What do you know about the answers to the number sentences you have made?**

To support the child here, you could ask them to make a picture of what these division sentences mean.

$$12 \div 3 = 4$$

You can show this with an array, or as hops of three back from 12 along a number line, or twelve biscuits into packs of three.



- ? **What picture could you make to show three divided by twelve?**

Help the child to read the sentence 'three divided by twelve'.

'Suppose we thought of three biscuits divided into twelve pieces. Each one of those pieces is going to be quite small (a quarter of a biscuit) so the answer to three divided by twelve is different from the answer to twelve divided by three!'

- ? **Show me the operation cards that we can use to reverse number sentences and still get the same answer. (+ and ×)**

- ? **Which operation cards are the ones where reversing the number sentence gives a different answer? (− and ÷)**

Finish with cards 3, 6 and 18, asking the child to make a division calculation that would go with $3 \times 6 = 18$.



- ? **Using any two numbers, write some calculations that are the same the other way around, for example, $17 + 15$ is 32 and so is $15 + 17$. Which operations don't work with numbers the other way around?**

Spotlight 1

Assumes that the commutative law holds for division also, for example assuming that $15 \div 3 = 5$, so $3 \div 15 = 5$

Opportunity for: exploring a real-life context

Tangerine arrays

Time 15–20 minutes

Resources

- Real or modelling material tangerines
- Counters or cubes
- Number cards (Resource sheets 1 and 2)

Key vocabulary

- | | |
|------------------|--------------------|
| other way around | count back |
| array | divided into three |
| multiplied by | fraction |
| divided by | |

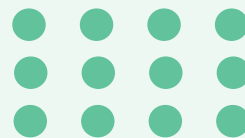
Teaching activity

‘Today we are going to make lots of number sentences about these tangerines. We are going to work with division cards only, and we are going to make number sentences that will always mean that the tangerines are not cut up.’

‘Let’s start with twelve tangerines and a 12 number card.’

? Can you choose two number cards that we can use to divide twelve into equal groups?
(6 and 2, or 3 and 4 are more useful in this context than 12 and 1.)

If the child is unsure about choosing numbers, you could lay out the tangerines in an array, or, instead of constantly handling real tangerines, you might want to make the array with counters.



Ask the child to describe the array.

‘There are three groups of four tangerines or four groups of three tangerines.’

? So can you choose two number cards now to go with this array? (3 and 4)

? Can you make two division number sentences with twelve, three and four?

If the child cannot do that, or is confused about the order of the number cards, help them by making one number sentence and reading it out: $12 \div 3 = 4$

'Twelve divided by three makes four groups.'

(Saying 'how many threes go into twelve' is potentially confusing, so keep away from 'goes into'. 'Three into twelve' is another confusing phrase.)

? Do we need to cut up any of the tangerines to divide twelve by three? (No)

'Show me how this division number sentence matches with the array.'

If the child can't relate the number sentence to the array, use the array to show how the twelve can be divided by three to give the answer four. Let the child split up the array and talk you through:

$3 + 3 + 3 + 3$ makes 12 and 3×4 makes 12.

If you start at 12 on a number line, you can hop back in threes.

$12 - 3 - 3 - 3 - 3 = 0$, so there are four threes in twelve.

Twelve divided by three makes four.

? Can we make a different number sentence with these same three numbers but still not have to cut up the tangerines?

If the child is finding that difficult, go back to the array and ask them if there is another way of looking at the array of twelve.

? What if we see the array the other way around?

You can encourage the child to either move the array, or move around the table so that they see the array as four rows of three.

Help them to make the number sentence:

$$12 \div 4 = 3$$

'Twelve divided by four makes three.'

'This relates to the array that is four rows of three.'



If the child wants to split the twelve into different numbers, for example seven and five, remind them that with multiplication and division we make equal groups. Seven and five are not equal groups.

If the child makes a sentence such as $3 \div 12 = 4$, or $3 \div 4 = 12$, ask them to think of the picture which those sentences would make.

? Do these number sentences go with our array of tangerines? (They don't.)

? Can you read those sentences to me?

' $3 \div 12 = 4$ is three divided by twelve, so if it was tangerines and we had three tangerines to divide between twelve people, we would have to cut the tangerines – and the answer wouldn't be four! The answer would be a fraction, just a little bit of a tangerine.'

? Can you change the cards around so that we can use all three numbers and it makes a division sentence where we don't have to cut up the tangerines?

If the child doesn't write sentences such as $3 \div 12 = 4$, you need to mention it to them anyway to make sure that they have considered calculations like this.

? Can you use these same three cards to make a division number sentence that would mean we would have to cut up the tangerines?

? Which number would we start with that would mean that we didn't start with many tangerines?

' $3 \div 12 = 4$ is three tangerines to be divided by twelve so we would need to cut the tangerines. The answer isn't four, so we must be very careful when we are writing division calculations that "go" with an array.'

If you think the child might be able to cope with it, you could ask them to draw a picture of what three tangerines divided by twelve might look like.



'If you cut them into twelve bits for twelve people, each person would just get a quarter of a tangerine. So the answer isn't four, it is less than one whole tangerine, just a fraction, so $3 \div 12 = 4$ is wrong.'

? What did you learn today about rearranging the numbers in division calculations?

Spotlight 2

Assumes that the commutative law holds for division also, for example assuming that $15 \div 3 = 5$, so $3 \div 15 = 5$

Opportunity for: developing mental images

More than one, less than one

Time 15–20 minutes

Resources

- Sweets or counters
- Bead string
- Number lines

Key vocabulary

- | | |
|------------------|---------------|
| other way around | hops of |
| groups of | fraction |
| divided by | more than one |
| count back | less than one |
| count forward | |

Teaching activity

‘Today we are going to do some division calculations and we are going to see which ones give us answers of more than one and which ones give answers of less than one.’

? What do I mean by a number ‘more than one’? What about a number less than one? (*Talk about a fraction of a whole apple – a bit that is less than one whole one, maybe a half or a quarter.*)

? What picture do you have in your head if I ask you to think about twenty divided by four?

‘You can use anything on the table to work that out, or you can draw the picture that is in your head.’

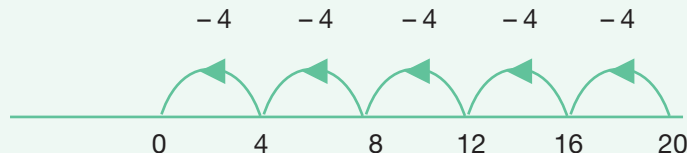
If the child seems stuck, ask them to read the sentence $20 \div 4$, ‘twenty divided by four’.

? Shall we start with twenty of these sweets or do you want to use a number line or a bead string?

If the child starts sharing ‘one for me, one for you’, accept this for now, but make a note to move them on from that later.

If the child shares the sweets into four groups of five, ask the following question.

? Is there another way to divide twenty by four? What if we put the sweets in groups of four? How would we show that on a number line?



Help the child to see that we can start at 20 and count back in steps of four. Five steps of four get us to zero. Or we could count up from zero in steps of four until we get to 20.

? So what is the answer to twenty divided by four?

If the child needs more support, make an array of five by four and show how it can be divided into five groups of four.

‘So twenty divided by four is five. That calculation gives us an answer of more than one.’

Record the calculation with cards or on paper.

$$20 \div 4 = 5$$

This answer is more than one.

? What if we reverse the twenty and the four?

$$4 \div 20 =$$

? Can you read that number sentence? (Four divided by twenty.)

? What picture do you have in your head?

Understandably, some children might be quite confused at this!

? How many sweets does that tell us to start with?

Explain that it means four sweets divided into twenty!

? If we divided four sweets into groups of twenty is each person going to get a whole sweet?

‘If we write the division this way around, the answer will be less than one.’

? What did you learn today about what happens when you change the numbers round in divisions?

‘Write a division with the numbers fourteen, two and seven so that the answer will be more than one.’

$$14 \div 2 = 7 \text{ or}$$

$$14 \div 7 = 2$$

If the child makes an error, note that for next time.

Spotlight 3

Assumes that the commutative law holds for division also, for example assuming that $15 \div 3 = 5$, so $3 \div 15 = 5$

Opportunity for: making decisions

.....

Tricky treats

Time 15–20 minutes

Resources

- Number cards (Resource sheets 1 and 2)
- $\times$, $\div$ and $=$ cards (Resource sheet 8)
- Cubes

Key vocabulary

- | | |
|-----------------|------------|
| other way round | divided by |
| array | count back |
| multiplied by | hop back |

Teaching activity

‘I’m going to play a game with you. I’m going to show you some number sentences and I might pretend to read a number sentence wrongly to trick you. You have to tell me if I am reading a number sentence wrongly or if I have put the wrong answer.’

Show two each of the cards 4, 5 and 20, and $\div$ and $\times$ operation cards. Make these two number sentences: $4 \times 5 = 20$ and $5 \times 4 = 20$.

? Am I tricking you?

? Are these number sentences both correct?

Both of those number sentences are right and give the same answer.

Make two division number sentences using the cards to display them:

$$20 \div 4 = 5 \quad 4 \div 20 = 5$$

? Are these number sentences both correct? Explain how you know.

? Can you make a picture of these sentences in your head and sketch it?

If the child cannot see that $4 \div 20 = 5$ is incorrect, support them in making pictures either on paper or with cubes, and ask them to read the sentences carefully.

‘ $20 \div 4 = 5$ can be read as twenty divided by four. That makes five hops of four along a number line, or an array that is four rows by five columns.’

‘ $4 \div 20 = 5$ can be read as four divided by twenty.’ (You could try reading it as twenty divided by four to see what reaction you get.)

‘Four cakes divided up into twenty pieces. The answer to that isn’t five. The pieces would be small and the answer would be less than one. I was tricking you. Four divided by twenty equals five is wrong.’

? Can we put the three numbers, four, five and twenty, in a different order to make a correct number sentence?

Support the child to re-order the cards to: $20 \div 5 = 4$ or $20 \div 4 = 5$.

‘Let’s make some tricks for the rest of the class. Let’s make some divisions that are right, and some that are wrong, and we will see if they can work out which ones are wrong.’

(50, 25, 2)

(30, 10, 3)

(35, 7, 5)



? Are these right?

$$50 \div 2 = 25$$

$$30 \div 10 = 3$$

$$7 \div 35 = 5$$

$$2 \div 50 = 25$$

$$10 \div 3 = 30$$

$$5 \div 35 = 7$$

Spotlight 4

Assumes that the commutative law holds for division also, for example assuming that $15 \div 3 = 5$, so $3 \div 15 = 5$

Opportunity for: linking mental images and language

Draw the picture

Time 10–15 minutes

Resources

- Several sets of number and symbol cards (Resource sheets 1, 2 and 8)
- Cubes

Key vocabulary

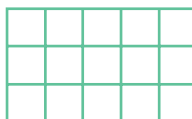
- | | |
|-----------------|------------|
| other way round | divided by |
| array | count back |
| multiplied by | hop back |

Teaching activity

‘Today we are going to do some drawings of pictures from our heads that go with number sentences, to help you to understand more about division and multiplication.’

? If we have the cards 3, 5 and 15, which multiplication sentence could you make?

Support the child to make $3 \times 5 = 15$.



? Can you put the numbers in a different order but keep the sentence correct?

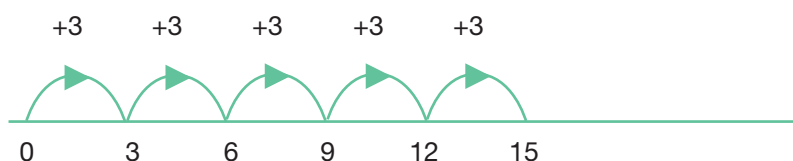
Help them to make $5 \times 3 = 15$.



? What is the picture that is in your head to go with $5 \times 3 = 15$?

While the child draws, you could draw an image as well, perhaps one that the child rarely draws or uses.

‘Five hops of three, so this image is three multiplied by five, or three five times.’



‘Five rows of three, or look at it the other way round to make three rows of five.’

Accept a range of drawings. (A really weird one might just give you insight into what the child is thinking!) Stress the point that you only want pictures that go with five, three and fifteen.

If the child makes something like 15×3 , show how that doesn't fit with the pictures.

Change the cards around again to make the two different multiplication sentences, making sure that the child can match the sentences to the various pictures.

Can you use two division signs to make two division sentences that are right and that fit with our pictures?

? What is it that we can do with multiplication calculations that doesn't work with division?

Help the child to read the division sentences carefully, so that $15 \div 3$ can be read as 'how many threes make fifteen', or 'fifteen divided by three'. To fit with the number line hops, try to move children from saying 'shared between' to saying 'divided by'.

If a child makes something like $3 \div 15 = 5$, help them to read it.

? Does it fit with any of our pictures? (It shouldn't!)

Clarify that this sentence means three divided by fifteen, or three divided into fifteen.

'This does not fit with the pictures we have been using. Each part would be just a fraction – less than one.'

? Explain to me what goes wrong if we try to swap numbers around in a division calculation.

? Can you give me an example to show me what you mean?

? Can you draw a picture to help me to understand?

? What helped you to learn something new today?

Spotlight 5: a learning check

Assumes that the commutative law holds for division also, for example assuming that $15 \div 3 = 5$, so $3 \div 15 = 5$

Opportunity for: discussing and explaining

.....

Fractions in the bin

Time 15–20 minutes

Resources

- Two or more players
- Counters or rewards
- *Fractions in the bin game board* (Resource sheet 40)
- *Fractions in the bin game cards* (Resource sheet 41)

Check: does the child use key vocabulary?

- | | |
|-----------------|------------------------------|
| other way round | divided by |
| array | count back |
| multiplied by | divided into groups of three |

Teaching activity

Note: Resource sheet 41 needs to be turned to landscape format to match these instructions.

The top three rows of cards go with the pictures and there is one blank card that you can use to challenge a child to write another card.

The cards in the fourth row are reverses of the arrays, for example 9×2 isn't a picture of the top picture on the board unless you view it from the side so that it becomes nine rows of two, nine two times, or nine multiplied by two. You might want to keep these cards for when children are more confident.

None of the cards in the fifth row matches any of the pictures, so again, you might want to keep these for later.

'We are going to play a game, **Fractions in the bin**, with cards on this board. Some of the cards go with the pictures on the board and some don't. So 2×9 would go with this picture, but $9 \div 18$ won't go with that picture so it goes in the bin.'

Put out just the cards you want to play with (probably just the first three rows at first) face up on the table, and a board for each pair of players. Give players a moment to look at the pictures and to count rows, and so on.

How to play

1. Each player starts with two rewards.
2. They take turns to take one card, read it carefully and place it near the right picture. If everyone agrees that they are right, they win a reward.
3. If they are wrong, they can think again about it and try to put it in the right place. If they are still wrong, they have to give one of their rewards back!
4. If a card doesn't match any of the pictures, this card is put in the bin. The player will win a reward if they are right that it doesn't match a picture.
5. The winner is the player with the most rewards at the end.

Variations

- Each child has their own game board and a set of cards (maybe five each). A timer is set and they try to put their cards in the right places as quickly as they can. When they have placed them all, a note is taken of the time and, the next time they do the activity, they race themselves to see if they can do it faster. For every wrongly placed card, ten seconds is added to their time!
- The players make their own game board of pictures and their own set of about ten cards, right ones and wrong ones. They could include written questions such as 'How many twos make eighteen?' They then swap sets of cards and race to put them in the right place. (This needs close supervision to make sure that the cards and pictures make sense!)

? Explain to me how you knew that picture matched that card.

? Why does that card go in the bin?

? Can you read that card and tell me what picture it makes in your head?

Learning outcomes

By the end of this set of activities children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- understand that division calculations give a different answer if the numbers are reversed;
- write a division calculation that links with a multiplication calculation;
- understand that multiplication number sentences can be reversed and still give the same answer.

Writes a remainder that is larger than the divisor, for example $36 \div 7 = 4$ remainder 8

Opportunity for: developing mental images

Resources

- Interactive Teaching Program (ITP) – *Grouping*

Key vocabulary

how many times?	take away
group	divided by
remainder	

Teaching activity

Time 20 minutes

Explain to the child that they are going to think carefully about division, especially the remainders, and find out exactly what to do when there is an amount left over.

Show the ITP set at $20 \div 4 =$

Revise the language of division, encouraging an understanding that $20 \div 4$ is the same as 'How many times can I take four away from twenty?' or 'How many fours are there in twenty?'.

Ask the child to collect groups of four by clicking and counting objects. Discuss how the objects are grouped on the number line. The child is finding out how many times they can take a group of four away. The number line is modelling how many times four can be added to make twenty.

Reinforce that the child must keep going until they have collected as many groups of four as possible.

Reset the ITP to $23 \div 5 =$ and repeat the activity, but discuss the remainder using the ITP model to demonstrate:

The '3' cannot be changed into a group of five. They are just 'left over'.

Reset the programme to show $21 \div 3 =$

? How many times do you think we can take a group of three away from twenty-one?

Check the answer using the ITP.

Work through more examples, including calculations that will leave a remainder, for example:

$$32 \div 10 =$$

$$14 \div 4 =$$

$$22 \div 8 =$$

Model an error, setting the ITP to show $30 \div 4 =$

Share your prediction that the answer will be six with a remainder.

? Do you agree with my prediction?

? Can you tell me exactly what the remainder will be/how many will be left over?

Model the error by collecting six lots of four and, with the child, count how many are left.

Discuss if that is the remainder.

? Have we collected as many groups of four as we can?

Use the ITP to demonstrate that we can make another group of four and the answer to $30 \div 4$ is 7 remainder 2.

Model another error, for example $21 \div 4 = 4$ remainder 5.

? If I am dividing by four, can I have a remainder of five?

Ask the child to use the ITP to show you the correct answer and to explain why you were wrong.

Work through some more examples together, encouraging the child to explain the remainders.

? If you are dividing by three, could you ever have a remainder that is more than three?

? If you are dividing by four, could you ever have a remainder of four?

? If you are dividing by five, what possible remainders can I have?

? What is important to remember for next time?

Spotlight 1

Writes a remainder that is larger than the divisor, for example $36 \div 7 = 4$ remainder 8

Opportunity for: reasoning about numbers

Eggs in sixes

Time 10–15 minutes

Resources

- Six egg boxes (six-egg size)
- Twenty-six eggs (or objects to represent eggs)

Key vocabulary

how many times? take away
group divided by
remainder

Teaching activity

‘Today we’re going to learn a bit more about remainders, and how we cannot have a remainder bigger than the divisor – the number you are dividing by.’

Outline a problem:

‘An egg box holds six eggs. If there were nineteen eggs I think we could fill two egg boxes and have some left over.’

? Do you agree? How could we check the answer?

Set out the nineteen eggs and collect a group of six to put in one box. Repeat with another group of six.

Discuss with the child: ‘We had nineteen eggs and needed to group them in sixes to fit in the box. We filled two boxes and have some eggs left over.’

? What was the calculation we carried out?

‘Nineteen divided by six equals two with some eggs left over.’

? Are you sure we couldn’t fill another box?

Ensure that the child remembers you always keep going until you can’t make another group.

Agree that one more box could be filled and we would have one egg left over.

Show the recording of the calculation that was carried out:

$$19 \div 6 = 3 \text{ remainder } 1 \text{ (or } 19 \div 6 = 3 \text{ r } 1)$$

Relate this calculation back to the story.

Repeat the activity with twenty-six eggs, reinforcing the key point that you cannot have a remainder larger than the divisor (the number you are dividing by) because you can still take away another group.

? Tell me what you think was the most important thing that we learned today.

Spotlight 2

Writes a remainder that is larger than the divisor, for example $36 \div 7 = 4$ remainder 8

Opportunity for: solving real-life problems

Football

Time 15–20 minutes

Resources

- Eight footballs
- Money – preferably real coins and notes (up to £44)

Key vocabulary

how many times?	take away
group	divided by
remainder	cost
left over	

Teaching activity

‘We’re going to do a money problem today, so that you will get even better at working with remainders when we divide.’

Outline a problem:

‘Footballs cost £5 each. I have £32 and want to buy as many footballs as I can.’

‘I think I will only be able to buy five footballs but there will be some change.’



? Do you agree with my calculation?

Enter into a role-play situation.

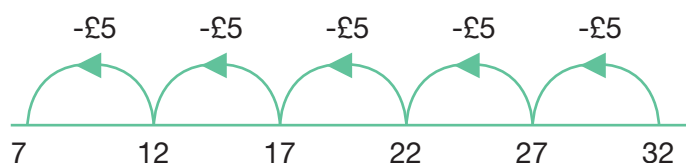
Suggest that the child is the sports shop assistant and that you are the customer. Explain that you would like to buy as many footballs as you can, but only have enough money to buy five. Give the assistant £5 and take one football. Repeat, exchanging one ball for each £5 until you have five balls.

Discuss with the child the fact that you have £7 left over.

? Have I bought as many footballs as possible?

In my change can you see another group of £5?

Remember I had to buy as many balls as I could.



After agreeing that you could get six footballs and have £2 left over, look at the corresponding calculation:

$$32 \div 5 = 6 \text{ r } 2$$

‘We found out how many times we could take £5 away from £32.’

Repeat the activity, starting with £44.

? How many footballs do you think I will be able to buy with £44?

? Do you think there will be any money left over?

Look at the calculation together after working through the activity.

? Did you enjoy working with money today? What did it help you learn about?

Spotlight 3

Writes a remainder that is larger than the divisor, for example $36 \div 7 = 4$ remainder 8

Opportunity for: reasoning about quantities

.....

Party punch

Time 15–20 minutes

Resources

- Two mini whiteboards and marker pens
- 2-litre jugs
- Counters

Key vocabulary

- | | |
|-----------------|-----------|
| how many times? | remainder |
| two litres | left over |

Teaching activity

‘Today we’re going to think about a problem where there is going to be a remainder. We mustn’t let the remainder get bigger than the divisor.’

Share a problem:

‘I have mixed a tub of party punch for the weekend.

In total there are 19 litres.

I need to pour this into 2-litre jugs to set out on tables.

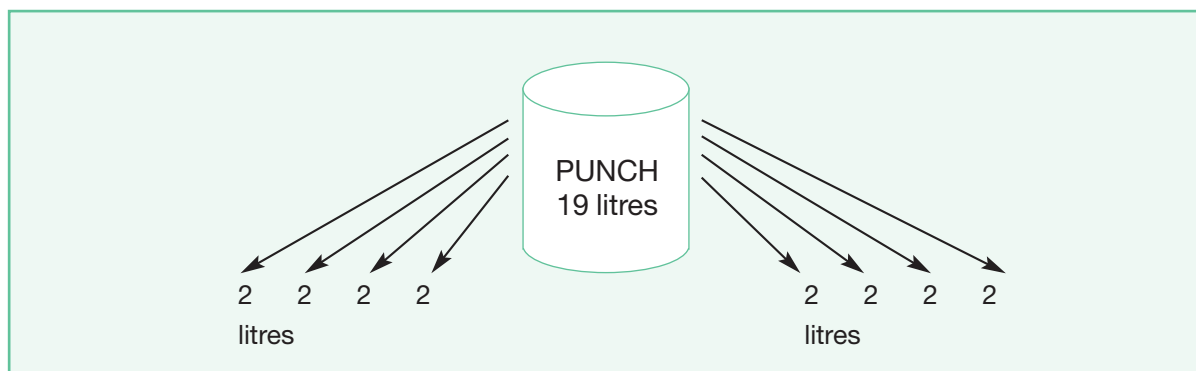
I have nine tables altogether but I am worried that I will only be able to fill eight jugs.’

? How many times do you think I will be able to fill a 2-litre jug out of my nineteen litres of punch?

? Explain to me how you worked that out.

Explore with the child how to use drawings/jottings to support thinking if you don’t have the apparatus.

‘Let’s draw the tub of punch. Now we’ll draw what we would do next. I thought I could fill eight jugs.’



Count in twos with the child and agree that we have filled eight jugs and used up 16 litres.

? How many litres will still be in the tub?

If necessary, set out nineteen counters and take away sixteen, two at a time.

? Is three litres the remainder? Could we fill another jug?

Model and talk through the calculation that was used to solve the problem:

$$19 \div 2 = 8 \text{ r } 3.$$

‘We can’t have a remainder of three litres because we could fill one more jug.’

The calculation should be $19 \div 2 = 9 \text{ r } 1$.

Reinforce that you can never have a remainder that is bigger than the divisor.

? Can you tell me why you can't have a remainder bigger than the divisor?

Spotlight 4

Writes a remainder that is larger than the divisor, for example $36 \div 7 = 4$ remainder 8

Opportunity for: reasoning about numbers

.....

Is it right?

Time 15–20 minutes

Resources

- Cubes
- Interactive Teaching Program (ITP) – *Grouping*
- Prepared series of calculations:

Key vocabulary

- how many times?
- group
- remainder
- take away
- divided by

A

$24 \div 5 = 5 \text{ r } 4$
$24 \div 5 = 4 \text{ r } 4$
$24 \div 5 = 3 \text{ r } 9$

B

$30 \div 8 = 4 \text{ r } 2$
$30 \div 8 = 2 \text{ r } 14$
$30 \div 8 = 3 \text{ r } 6$

C

$18 \div 5 = 3 \text{ r } 3$
$18 \div 5 = 2 \text{ r } 8$
$18 \div 5 = 3 \text{ r } 2$

Teaching activity

‘Today we are working on the computer again, looking at grouping and the remainders that are left over.’

? Can you remember something important about remainders?

Look together at card A and set the ITP to $24 \div 5 =$

? Can you tell me which of the answers is the correct one?

Talk to me about each of the calculations.

Use the ITP model to demonstrate grouping sets of five.

Stop after grouping three ‘lots of 5’.

? How many counters are left?

? Can we have nine as a remainder when we are dividing by five?

Work through each of the examples in the same way until the correct answer has been modelled.

Reinforce that the remainder can never be the same as or bigger than the divisor.

Work through the other series of calculations in the same way.

Encourage the child to work with the ITP but to predict, explaining the correct answer from the set, as they work.

Record some of the ‘silly’ number sentences.

Spotlight 5: a learning check

Writes a remainder that is larger than the divisor, for example $36 \div 7 = 4$ remainder 8

Opportunity for: discussing and explaining

Silly number sentences

Time 15–20 minutes

Resources

- Cubes
- Bag
- Empty number lines
- At least two children
- Prepared series of number sentences

Check: does the child use the key vocabulary?

how many times? take away
group divided by
remainder

$$15 \div 5 = 4$$

$$17 \div 4 = 5$$

$$13 \div 3 = 4$$

$$15 \div 5 = 3$$

$$17 \div 4 = 4 \text{ r } 1$$

$$13 \div 3 = 3 \text{ r } 4$$

$$15 \div 5 = 2 \text{ r } 5$$

$$17 \div 4 = 3 \text{ r } 5$$

$$13 \div 3 = 4 \text{ r } 1$$

Teaching activity

‘Today we are going to play a game, **Silly number sentences**, and you are going to decide which ones are silly and which ones are sensible.’

? Tell me what you have remembered about the size of a remainder when we have been dividing.

Prepare several number sentences – some right, some wrong – as in the resource list.

The children could work cooperatively in pairs or teams, or they could compete against each other.

You might want to play the game where the players can shout.

Put all the number sentences in the bag so that you can pull them out one at a time, and so that each player can see the sentence at exactly the same moment.

How to play

1. Take a number sentence out of the bag and show it to everyone.
2. If it is a silly sentence, for example $18 \div 5 = 2 \text{ r } 8$, everyone shouts ‘silly!’.
3. If it is a sensible sentence, for example $11 \div 2 = 5 \text{ r } 1$, everyone shouts ‘sensible!’.
4. The players score ten points every time they get one right.

If a child needs more help, use cubes or number lines to work out the sentences and put them in two groups, silly and sensible, so that you can help the child to reflect on what is the same about the silly sentences.

? What makes a silly sentence silly?

Variations

- The children could compete and win a card if they shout first.
- The children could make up sets of three number sentences themselves to give to a friend to play the game.



? What do you notice about this: $50 \div 7 = 6 \text{ r } 8$?

Learning outcomes

By the end of this set of activities children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- decide whether a remainder is within the range appropriate for a particular division calculation;
- make decisions about how to treat a remainder based on the context within which it occurs;
- explain how they know that a particular remainder is a reasonable one.

Notes

Notes

Discards the remainder; does not understand its significance

Opportunity for: understanding and communicating mathematical ideas and symbols

Resources

- Cubes
- Number lines
- Paper plates and pretend sandwiches

Key vocabulary

divided by left over
divided into remainder
divided between hops of
shared between

Teaching activity

Time 10–15 minutes

‘We are going to do some division today with these sandwiches, and we are going to think carefully about what we do when we have some items left over, a remainder.’

? Can you tell me what you think having a remainder after doing division means?

Build on what the child says.

? Can you show me what you mean if we had eight sandwiches and we had to put them three to a plate for a party?

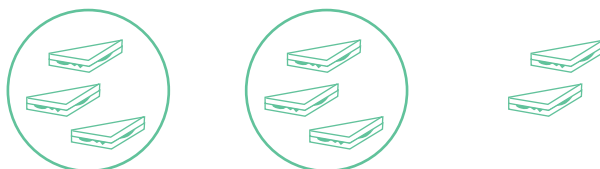
Support the child putting three sandwiches on each plate, saying ‘eight divided by three’.

? Can you show me how you would write down eight divided by three?

If the child doesn’t know, help them to write $8 \div 3$ as you say ‘eight divided by three’. You will need to give them a few more examples to write down, such as:

seven divided by three;
sixteen divided by five;
nineteen divided by two.

Make sure that they can write these in symbols and read them back to you in words. Aim for ‘divided by’ as the main words you use, as the example of the sandwiches was a grouping model.



Come back to the calculation eight divided by three and talk through the sandwiches on the plates. Try to end up with a calculation.

$8 \div 3 = 2$ complete plates of three sandwiches with two sandwiches left over,
or $8 \div 3 = 2$ and a remainder of two sandwiches.

You might want to show children this recording:

$8 \div 3 = 2$ remainder 2

? If we have eight sandwiches and each plate has to have three sandwiches, how many complete plates can we make? So how many people at the party can have a plate of three sandwiches?

? What about these two sandwiches left over? What could we do with them?

Make a note of responses. (You will be adding to this list throughout this set of activities.)

We could:

- eat them;
- divide them up into smaller bits and share them around...

Clarify that you probably wouldn't just leave the sandwiches to go hard and stale. You would do something with them.

? Can you choose something on the table to show me how you would divide seventeen muffins by three?

? How many muffins will you put in each group if they were 'divided by three'?

Build on where the child is with their thinking and record:

Seventeen divided by three.

Seventeen muffins put into groups of three.



Seventeen divided by three is five groups of three and two muffins left over.

Five people can have three muffins each.

? What might we do with the two muffins left over?

If the child needs more help, give some more practical examples.

'What if we had sixteen sandwiches for the party and we wanted to give people five sandwiches each. Let's write the division calculation sixteen divided by five.'

$$16 \div 5 =$$

Let the child work this out any way they want (cubes, number lines or moving sandwiches onto plates) and observe carefully what they do. If they share out with a 'one for you, one for me' method, let them continue with this but make a note that you want to move them on from it during this work on division.

If they share they might end up with five groups of three and one left over, because they were using a dividing between five model rather than dividing by five.

You could build on that and ask them what they will do with that one left-over sandwich.

Add their ideas to your list of their suggestions of practical things they could do with the remainders based on the context.

‘Let’s put out some plates and put five sandwiches on each plate, that is sixteen divided by five.’

? How many fives do you think there are in sixteen?

Try to get to the response, ‘There are three fives in sixteen and one left over.’

(You can support a child with a number line to work this out.)

‘Look at our list of things we thought we could do with left-over sandwiches. Mostly we thought that we wouldn’t just leave the sandwiches to go hard and mouldy! We can’t just ignore the left-over bits after a division!’

? What did you learn today?

Spotlight 1

Discards the remainder; does not understand its significance

Opportunity for: solving problems in a real-life context

.....

Tom's tickets

Time 15–20 minutes

Resources

- Cubes
- Number lines
- Money

Key vocabulary

divided by	left over
divided into	remainder
divided between	hops of
shared between	

Teaching activity

'We are going to do some more about remainders today, and this time we are going to work with money.'

grouping model

? If Tom had £15 and tickets for the disco were £6 each, how many tickets could he buy?

Let the child work this out any way they want and observe carefully what they do.

'Tell me what you are doing as you do it.'

Support the child using the key vocabulary.

? What division calculation would you write for finding out how many £6 tickets Tom could buy?

If the child doesn't know, help them to record in some way that makes sense to them.

A number line can show remainders clearly.

'We need to hop back from 15 in steps of six.'

Two hops of £6



'We have £3 left over.'

? If we weren't thinking about pounds and tickets, how would you work out fifteen divided by six?

Try to end up with $15 \div 6 = 2$ remainder 3.

'So Tom can buy two tickets and he has £3 left over.'

? If you were Tom, what would you do with that £3 left over?

Record the responses, such as:

- Save the money for something else.
- Ask Dad for £3 so that another ticket can be bought.
- Give the money to a friend so that they have enough to buy a ticket for themselves.

? At the disco, cold drinks are eighty pence each, so how many can Tom buy for £3?

grouping model



? Can you make an estimate first? About how many drinks? Just a few, or more than five?

Encourage the child to see that not many drinks can be bought, because 80p is quite close to £1.

Let the child work out the exact answer any way they want and build on what they say and do.

If the child doesn't know what to do, counting back in groups of 80p might help.



? How much money does Tom have left over?

? What might he do with that money?

(Note: You might want to note how familiar the child seems with money. For some children, putting a task in terms of money can help enormously because they are used to handling money. However, some children have had so little experience with shopping that money tasks still confuse them. They will need to do further problem solving with money and play money games. See Spotlight 5.)

? Can you tell me what you have learned about remainders today?

Explain that you are going to keep their list of ideas for what they could do with remainders and add to it next time.

Spotlight 2

Discards the remainder; does not understand its significance

Opportunity for: reasoning about numbers

Let's go to the beach!

Time 15–20 minutes

Resources

- Cubes
- Base 10 rods
- Number lines
- Calculator

Key vocabulary

- | | |
|-----------------|-----------|
| divided by | left over |
| divided into | remainder |
| divided between | hops of |
| shared between | |

Teaching activity

'Today you are going to pretend to be the after-school club leader, and you are planning a day out to the seaside in half-term for every child in the school. There are two hundred and forty children and we don't want to leave any of them behind.'

'Each coach can take fifty children. (There is also space for two adults because there are fifty-two seats, but leave that out of the calculation.)'

? How many coaches do you need to take all two hundred and forty children to the beach?

Let the child work this out in any way they want. Establish that the calculation is written down as:
 $240 \div 50 =$

? How many fifties are there in two hundred and forty?

Or how many groups of fifty can we make from two hundred and forty?

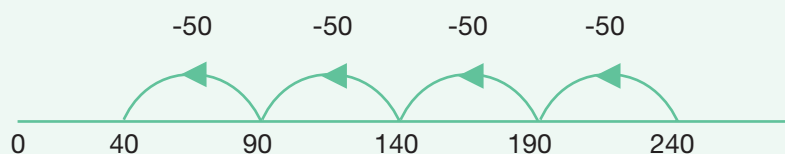
? Can you estimate this first?

If they can't, let them move on so you can see what they are thinking. If they look as if they are going to count out two hundred and forty cubes, you need to stop them and ask them if they can find a quicker way of doing it.

Two hundred and forty as twenty-four Base 10 rods would be better. These could be put into groups of fifty (five 10-rods).

Using a number line is a good method. They can use a calculator to check their numbers.

Count back in steps of fifty



? How many steps of fifty?

Alternatively, let them use a calculator from the start. It can lead them right into the trap you almost expect them to fall into!

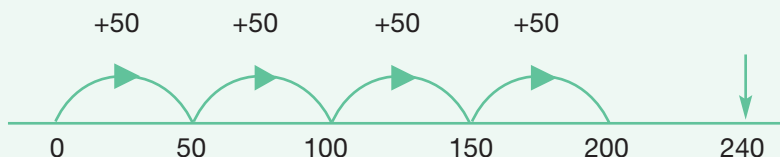
On the calculator $240 \div 50 = 4.8$, so if they tell you they need 4.8 coaches, this is the moment to ask them just what 4.8 coaches would look like!

Worked out on a number line, counting back from 240 in steps of fifty, there are four lots of fifty to get to 40 on the number line, then the left-over forty children need a coach as well.

If the child is confused about the larger numbers, you could do an addition calculation, maybe using 10-rods of some kind to support the large numbers.

' $50 + 50 + 50 + 50 = 200$. That is four full coaches.'

You can show this by counting up on the number line.



'Then there are another forty children.'

? What could you do if you were organising the trip and there were only forty children to put on the last coach? You won't just ignore them, will you? They don't want to be left behind!

? What could you do with the extra forty children?

Record some of the child's responses to add to the list of responses from earlier work.

Maybe there is...?

Or maybe...?

Remind the child of the list of their ideas of things to do with remainders and add to it their responses from today.

? Tell me what you learned today about remainders.

Spotlight 3

Discards the remainder; does not understand its significance

Opportunity for: making a connection between mathematical symbols and real-life contexts

.....

Number line remainders

Time 10–20 minutes

Resources

- Number lines

Key vocabulary

divided by	left over
divided into	remainder
divided between	hops of
shared between	count back

Teaching activity

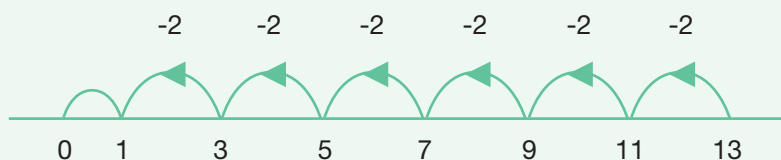
‘We are going to do some more division calculations that have a remainder, and we are going to see if we can make up some stories to go with the calculations.’

? Can you write down thirteen divided by two?

? How can you work that out on a number line?

Support the child writing $13 \div 2 =$ and saying ‘thirteen divided by two’. If they are unsure about drawing a number line for themselves, they can use a blank commercial line, but you might need to start them off by marking the 13 at the right-hand end.

Six hops of two and one left over



$$13 \div 2 = 6 \text{ remainder } 1$$

There are six twos in thirteen with one left over.

‘Let’s try to make a story to go with thirteen divided by two.’

If the child can’t think of a story, you could give them a starting point. For example, ‘Dan has thirteen Easter eggs and his mum says he mustn’t eat more than two a day.’

? For how many days can he eat two Easter eggs? (Six with one egg left.)

Build on the child’s own ideas where you can.

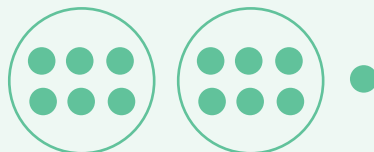
Apples, biscuits, sweets, muffins, pens and £1 coins could also be used to help the child to show how they know their story works.

? What could Dan do with the left-over Easter egg?

Develop the child’s ideas for how to deal with remainders and add these to their earlier list of ideas.

It is likely that some children might say, for thirteen divided by two (which is a grouping story), ‘Thirteen sweets shared between two children is six each and one left over’, making it a sharing story that does not fit with the number line jumps of two above.

The image that might fit with thirteen divided *between* two is:



grouping model

sharing model

These children need more experience with using the words ‘divided by two’ and ‘how many twos make thirteen’ and with using the image of a number line, counting back in a known step size (two in our example), rather than reading the division sign always as ‘sharing between’.

You can give some further examples of calculations that can be solved on a number line, focusing on ‘how many steps back’.

- ? How could you solve the problem seventeen divided by two on a number line?
- ? Can you make a number line drawing to show how many fours there are in twenty-one?

Spotlight 4

Discards the remainder; does not understand its significance

Opportunity for: solving practical problems and making decisions

Two and a bit

Time 10–20 minutes

Resources

- Cubes
- Number lines
- Money
- Apples, biscuits, or soft modelling material

Key vocabulary

- divided by
- divided into
- shared between
- left over
- remainder
- hops of
- count back

Teaching activity

‘You are getting good at remainders and today we are going to find some more remainders.’ Look together at the child’s list of suggestions for dealing with left-over items.

- ? **Supposing you and your friend had been washing cars to earn some money and you earned £2.55. How could you divide that equally between the two of you?**

Support the child, building on what they say and using coins (or pencil and paper jottings if the child chooses these) to clarify the division.

If the child cannot divide £2.55, you will need to spend time with coins, dividing money amounts such as £2, £3, £2.50, and so on, making sure they can recognise coins, can name them, and know how to both double and halve a simple amount of money.

Divide the £2.55 into two lots of £1.27 and show the 1p left over. Remind the child of odd and even numbers.

? What might the friends do with the 1p?

If the child needs more work with money, you could make up some more practical money divisions, such as £2.55 divided between three friends, and so on.

? Can you work out how a packet of eight sausages could be divided between a family of five people?

Talk the child through what some of the different outcomes could be. Encourage them to use modelling material or objects to look at the possibilities.

This could be a time to let the child record in their own way, perhaps with pictures that can be taken back to class and talked through with the rest of the class.

Eight divided between four would be two sausages each, but divided between five is less than two each.

If the child is having difficulties with this, you will need to do further practical examples with objects, until their language has developed and they can divide small numbers reasonably confidently, making a sensible decision about what can be done with different remainders.

Clarify with the child that remainders are important and we need to think about them carefully and decide what is best to do.

‘We can break a biscuit into halves, but we can’t tear a £10 note in half! We have to consider remainders carefully to fit with what we are dividing.’

? What shall we record from today that is important to remember?

Spotlight 5: a learning check

Discards the remainder; does not understand its significance

Opportunity for: discussing, explaining and reasoning about numbers

Hand over the beans

Time 5–15 minutes

Resources

- Cubes
- Number lines
- Dried beans
- Bag to contain number cards
- Number cards 1 to 50 (Resource sheets 1, 2, 3 and 4)
- $\div$ and $=$ cards (Resource sheet 8)
- *Multiplication grid* (for example, Resource sheet 35)
- At least two players

Check: does the child use key vocabulary?

divided by	left over
divided into	remainder
divided between	hops of
shared between	

Teaching activity

'We're going to play a game today called **Hand over the beans** to practise what we've learned about remainders.'

Use number cards to suit your children. You could have 12, 14, 15, 16, 18, 20, 21 in a bag and 2, 3, 4, 5, 6, 7, 8, 9, 10 on the table.

Some children might need to play the second variation of this game first if their knowledge of linking divisions to multiplication tables is not secure.

Put the bag with the larger numbers on the table, so the cards can't be seen, and put the 2, 3, 4, 5, 6, 7, 8, 9, 10 cards face up on the table and spread out (see variation on page 12).

How to play

1. Everyone starts with five beans. Someone will be bean banker and look after the rest of them.
2. Player 1 takes a card out of the bag, for example 15. They must select a smaller number from the table so that they can make a division calculation that does have a remainder. They can use cubes or number lines to work it out.
3. Player 1 must read and write their calculation and explain why they think they are right. If everyone agrees that they are right, they will win as many beans as there are in their remainder.
4. If the other players think player 1 is wrong in what they say or write, the other players can challenge them and must prove why player 1 is wrong. The other players can draw a number line, or look at the multiplication grid. If they do prove that player 1 is wrong, player 1 must give a bean to each of the other players!
5. The other players then take their turn to take a card from the bag and choose a smaller number from the face-up cards so that they make a division with a remainder.
6. The winner is the player with the most beans at the end.

? Can you explain to me why you chose that card to go with 21?

? How do you know you are right?

Variation

- Once the players have understood the game, help them to see that if they select their smaller number carefully, they can end up with larger remainders. For example, $15 \div 9 = 1$ remainder 6, giving a much larger number of beans to win than with $15 \div 2 = 7$ remainder 1.

? Could you have chosen another card to make your remainder larger?

? Could you change the number cards you play with so that you can make even bigger remainders?

- Put all the cards face down in two groups or piles. Take a card from each pile as the numbers to work with. (This is more reliant on luck but can also be quite a bit harder.)



This variation is an appropriate problem-solving task for a whole class.

- Play so that you don't have a remainder. If 18 is taken out of the bag, that player must choose 2 or 3 or 6 or 9 from the smaller numbers and make their division sentence, read it and explain it, showing that there is no remainder.

$$18 \div 6 = 3 \text{ and no remainder}$$

If the player is right, they win a bean. If they are wrong, they must give a bean to each of the other players.

Learning outcomes

By the end of this set of activities children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- understand a range of strategies to deal with remainders in real-life contexts;
- use a number line to show division and remainders;
- link a division calculation to a real-life example.

Does not recognise when a remainder is significant in the decision about whether to round up or down

Opportunity for: solving problems and making decisions

Resources

- Twelve 'mini' boxes of chocolate beans
- Real money – 10p and 1p coins
- Mini whiteboard and pens

Key vocabulary

how many times?	division
remainder	group
take away	spend
divided by	round

Teaching activity

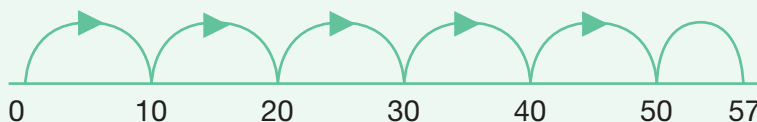
Time 20 minutes

'We are going to think carefully about what to do with the remainder when we are using division to solve problems.'

Look together at the calculation:

$$57 \div 10 =$$

? How many tens are there in fifty-seven?



Use jottings on a whiteboard to encourage the child to calculate the answer:

$$57 \div 10 = 5 \text{ r } 7$$

? Tell me, what do you think the numbers in the answer mean?

Encourage the discussion of five groups of ten and seven left over.

Look together at a real-life problem that uses the same numbers.

'Here are twelve mini boxes of chocolate beans that cost ten pence each. I have fifty-seven pence to spend.' (Check the amount of money together.)

? What do you need to do to work out how many boxes of chocolate beans you could buy for fifty-seven pence?

If the child has difficulty in finding a strategy to get started, support them to consider the problem in terms of:

? How many 10p coins could we add together to make fifty-seven pence?

and/or

? How many times can we spend (take away) ten pence?

Support the child to solve the problem practically, using the resources to support finding a solution.

Set out five 10p and seven 1p coins in a row.

Position one box of chocolate beans under each 10p coin, agreeing that we could have one box of chocolate beans for each 10p we spend.

? Can we buy another box of chocolate beans with the remaining seven pence?

Agree with the child that you could not buy a box of chocolate beans with the seven pence left, and that you can't buy part of a box. The seven pence would be left over (a remainder).

? Is the answer to our question five remainder seven?

? What do you think the answer is? How do you know?

Read the question again, emphasising that we are being asked: 'How many boxes of chocolate beans can we buy?'

We know we can only buy five boxes so that is our answer.

We do not have to say how much money is left over for this question.

Explain that when we solve problems we have to think carefully about what the question is asking and then 'round' our answer to make it make sense.

Repeat the teaching activity based on $95 \div 10 =$

Work through the calculation with the child, jotting on a whiteboard. Then use coins and the boxes of chocolate beans. Remind the child that we are thinking about the problem and have to work out how to use the result of our calculation to answer the question.

Spotlight 1

Does not recognise when a remainder is significant in the decision about whether to round up or down

Opportunity for: using mathematical symbols to communicate

.....

Jumping taking away

Time 10–20 minutes

Resources

- Twenty-eight 'eggs' (ping pong balls or similar) in a bowl
- Six egg boxes
- Mini whiteboard and pens

Key vocabulary

how many times?	division
remainder	group
take away	spend
divided by	round
how many jumps?	

Teaching activity

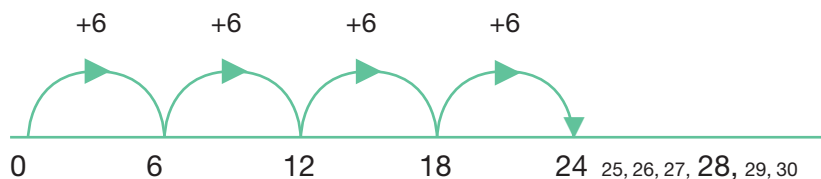
'Today we are going to be thinking carefully about what to do with the remainder when we have used division to solve a problem.'

Work together to calculate: $28 \div 6 =$

Use whiteboard jottings to support the child's thinking, encouraging the child to make a record of their thinking.

Remind them that we could think about this as ‘How many times could we take six away from twenty-eight?’, or as ‘How many times would we have to add six to make twenty-eight?’

Work along a number line in single jumps of six.



We have added six four times to reach 24.

? Can we fit in another jump of six?

? How many can we count between 24 and 28?

Discuss and agree that the answer to $28 \div 6 = 4 \text{ r } 4$.

Remind the child that we are thinking carefully about what to do with remainders when we are solving problems.

Look together at the following problem: ‘Here are twenty-eight eggs. I want to pack them into these egg boxes. How many egg boxes can I fill?’

? How many eggs will fit into one box?

? What is the calculation we need to do?

Encourage the child to see this as finding out how many times we can take groups of six eggs away from the twenty-eight eggs.

Work through the problem practically using six egg boxes and twenty-eight eggs.

Ask the child to count out six eggs and place them in egg boxes as they count.

If the child continues to count out eggs and place them in boxes after four boxes are full, stop them and return to the initial question.

? How many egg boxes can I fill?

Discuss and agree that the remaining eggs will not fill another box.

? The answer to our earlier calculation was four remainder four.

We have filled four egg boxes and we have four eggs left over.

How many boxes can we actually fill?

? What happens to the remainder for this problem?

Spotlight 2

Does not recognise when a remainder is significant in the decision about whether to round up or down

Opportunity for: reasoning with numbers

.....

Pack all the eggs

Time 10–20 minutes

Resources

- Twenty-eight 'eggs' (ping pong balls or similar) in a bowl
- Six egg boxes
- Mini whiteboard and pens

Key vocabulary

division	take away
how many times?	spend
group	divided by
remainder	round

Teaching activity

'We are going to be thinking carefully about what to do with the remainder when we have used division to solve problems.'

Quickly revisit the Spotlight 1 activity, reinforcing the key point that we rounded down from four remainder four to four in that problem, because the question was: 'How many egg boxes can I fill?'

Look together at a new problem that is very similar: 'I have twenty-eight eggs and would like to pack them all into egg boxes to keep them safe. Each egg box holds six eggs. How many egg boxes will I need to pack *all* my eggs safely away?'

? Do you think the answer will be the same as the answer to the last problem?

Assess the child's understanding as they explain their thinking.

Work through the problem in a practical way.

Ask the child to pack the eggs into boxes filling one box before moving to the next.

If they stop after filling four boxes because they know there are not enough eggs to fill another box, stop them and return to the initial question.

? How many egg boxes will I need to pack all my eggs safely away?

Discuss with the child how this is different from the last question and encourage them to pack the remaining four eggs into another box.

? How many egg boxes did we need?

Agree the answer is: 'I will need five boxes.'

Reinforce the need to think really carefully and to keep going back to check the question that you are answering.

? What do you think is the most important thing you must remember about finding answers when there is a remainder?

Spotlight 3

Does not recognise when a remainder is significant in the decision about whether to round up or down

Opportunity for: solving real-life problems

6c Y4 x / ÷

Pop concert

Time 10–20 minutes

Resources

- Mini whiteboard and pens

Key vocabulary

division	take away
how many times?	spend
how much?	divided by
remainder	round

Teaching activity

‘Today we are going to be thinking really carefully about what to do with the remainder when we have used division to solve problems. We will see how useful drawing/jotting can be in helping us to understand the problem.’

Read the problem together:

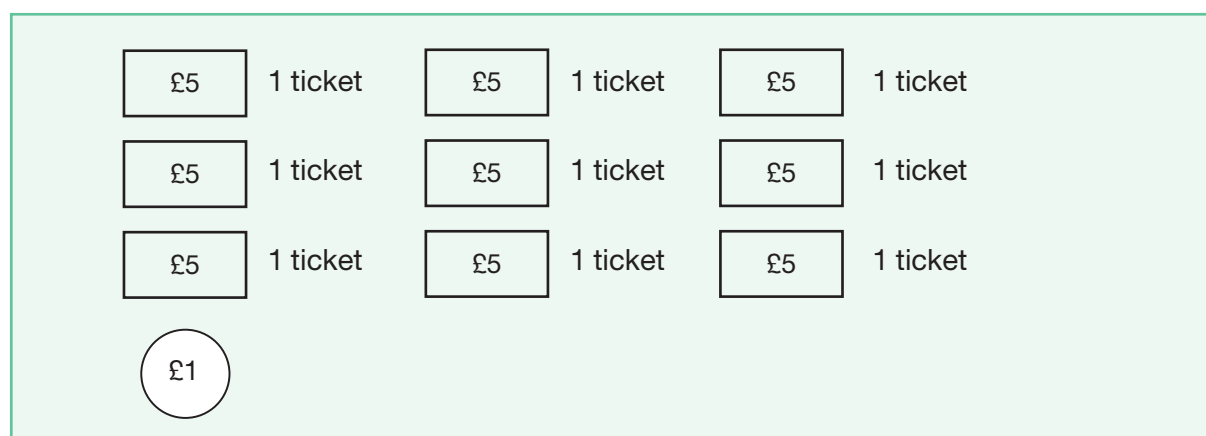
Tickets for a Top Pops concert are on sale at a special price of £5 each. I would like to take some friends. I have £46 altogether. How many tickets could I buy?

? Can you think of a way to use jottings or drawings to help you to understand how you could solve the problem?

Explain that a simple drawing can be very useful to help find out what calculation to do.

Support the child in making a diagram/drawing and using it to identify which calculation they need to do.

If they find producing a drawing of their own too challenging, offer something like the following as support, drawing the £5s one by one as the child counts up to find the nearest to £46:



? How many tickets can we buy with the £46?

? Is there any money left over (a remainder)?

? The question was: ‘How many tickets could I buy?’ What is the remainder and what happens to the remainder in this problem?

If the child suggests that the remainder is included in the answer, revisit the question and explain that the answer ‘We could buy nine remainder one tickets’ does not make sense.

It sounds as though we have one ticket left over, when it is actually £1 that is left over.

Remind the child that we have to think really carefully about deciding what to do with the remainder, and it is very important to go back to the question and look carefully again at our drawings to understand what we are working out.

Spotlight 4

Does not recognise when a remainder is significant in the decision about whether to round up or down

Opportunity for: solving practical problems and making decisions

Party bags

Time 10–20 minutes

Resources

- Six paper bags
- Selection of twenty-six small toys

Key vocabulary

division	divided by
share them equally	round
how many times?	left-overs

Teaching activity

Explain to the child that we are going to be thinking carefully about what to do with the remainder when we have used division to solve problems.

Read a problem together:

Liam is helping his mum to make party bags for his sister’s fourth birthday at the weekend. Six children will be at the party. Liam’s mum has twenty-six goodies altogether. She wants to share them equally between the six children.

How many goodies will each child get?

? What will you find helpful to start solving this problem?

Encourage the child to talk to you about what is happening and try to model it in some way. Support them in moving forwards from ‘the story’ to understanding what calculation is needed.

? Have you thought of a good way of getting started on this problem?

If the child has difficulty getting started, remind them that we can understand this as thinking about:

‘How many times can we take a group of six goodies from the bowl of twenty-six to put in the six party bags?’

Ask the child to use the toys and role play the task to help them understand the problem.

Support the child in recording the necessary calculation and their jottings to support their thinking.
Reread the question with the child, and ask them to look again at the bags and left-overs they have.

? Would it make sense to say that each child would get four remainder two goodies?

? Why do you think this?

Emphasise that for this question we round the remainder down and the correct answer is that each child will get four goodies.

? Why did we have to round down?

Spotlight 5: a learning check

Does not recognise when a remainder is significant in the decision about whether to round up or down

Opportunity for: explaining and discussing

Canoes

Time 10–20 minutes

Resources

- Mini whiteboard and pens

Check: does the child use key vocabulary?

division	take away
how many times?	divided by
group	round
remainder	

Teaching activity

Explain to the child that we are going to be thinking carefully about what to do with the remainder when we have used division to solve word problems.

Read through this problem together:

The school has taken a group of children on an adventure holiday. On Monday, they are going to try canoeing on the lake. Each canoe holds six pupils. Thirty-nine pupils will be in the group.

? How many canoes will be needed to take all the pupils on the lake at the same time?

Assess the child's ability to reason about the remainder by encouraging them to use the whiteboard to 'draw' a picture of the problem and to talk to you to explain their thinking.

Allow them to work through the whole problem without intervening so that you can be sure about what they understand.

If the child is still struggling to understand when to round up and when to round down, model a 'good' jotting that makes the decisions clear.

If they work independently and make the correct decisions ask a second question.

? How many canoes will be full?

Ask them to refer to their previous jotting to explain their answer to you.

? Tell me what you think you've learned about solving problems where you have to think about a remainder while you are finding an answer.***Learning outcomes***

By the end of this set of activities children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- carry out division calculations resulting in remainders;
- identify from the context of a problem how to deal with any remainder from the division.

Continues to subtract twos when calculating twenty divided by two without using knowledge that two multiplied by five equals ten

Opportunity for: developing mathematical strategies

Resources

- Coloured pens
- Number lines
- Bead string

Key vocabulary

- | | |
|-------------|------------------------|
| steps back | chunks |
| hops back | same amount altogether |
| take away | divided by |
| bigger hops | |

Teaching activity

Time 10–20 minutes

Explain to the child that today's activity will help them to do division calculations much more quickly.

? Can you show me how you would do the calculation twenty divided by two on paper?

Observe how the child does the calculation.

If they are not using some kind of subtraction method, you will need to start them off with a number line or a bead string method and show how this links to a paper and pencil method.



Twenty beads take off two at a time is:

$$20 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 = 0$$

What a slow way to work it out!

'We could take ten hops of two back from 20 to get to 0.'

Help the child to complete ten hops of two on a number line.

'So twenty divided by two equals ten.'

? Can you think of a quicker way of doing this subtraction?

? Can you think of any quicker way of splitting up the twenty beads to take them away in bigger chunks than just two at a time?

? What would we do on this number line to show a quicker way than subtracting two every time?

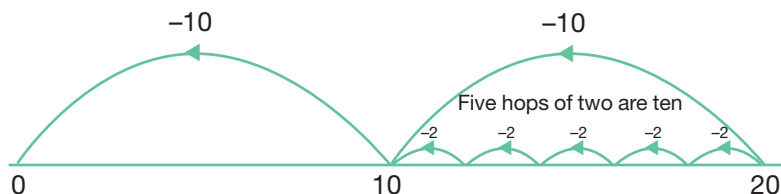
If the child can't work that out, show them how they could take bigger hops back.

'I'm going to show you a much quicker way than doing the ten hops back. I'm going to use something I know about multiplication to do it in fewer steps.'

'I know that two multiplied by five equals ten, so hops of two five times is ten. So I'm going to take an enormous step back and get from 20 all the way to 10. That has the same value as taking five little hops of two each.'

Let the child fill in the five small hops of two on your number line.

'One big hop of five lots of two is ten.'



Repeat, putting in five more little hops of two from 10 to 0.

'So I could do the division twenty divided by two very slowly by taking away two at a time, or I can use my multiplication tables and take away five lots of two all in one go to get to 10, then another five lots of two to get to 0.'

? How many hops of two have I taken away?

? How much did I take away altogether with those two big hops?

Support the child by talking it through again, perhaps going over the hops in different colours.

? When we did all these little red hops, how many were we taking away at a time? (Two)

? When we did the bigger and quicker green hops, what were we taking away each time? (2×5)

? So did we take away the same amount in the end? Explain it to me.

Use the number line again, or the bead string, to show that taking away five lots of two each time is quicker.

'We took away five lots of two then another five lots of two, so in the end we have the same answer. Twenty divided by two is five hops of two and another five lots of two. That is ten lots of two altogether: the same as our slower method.'

'Now I am going to hide the drawing we did and I want you to talk me through it as you do it again. Use the red pen for the little hops and the green pen for the big hops.'

? How many little hops of two are there from 20 to 0? (Ten.)

? How many big hops of $2 \times 5 = 10$? (Two.)

? How do you know?

$$20 \div 2 = 10$$

'Today we were finding ways to make division a bit quicker and we did that by taking bigger hops back on the number line.'

? What do you think you need more help with?

Spotlight 1

Continues to subtract twos when calculating twenty divided by two without using knowledge that two multiplied by five equals ten

Opportunity for: working systematically

7
Y
4
X
/
÷

Slow way, fast way

Time 10–20 minutes

Resources

- Coloured pens
- Number lines
- Bead string
- 100-square and/or *Multiplication grid 1* (Resource sheet 35)

Key vocabulary

- | | |
|-------------|------------------------|
| steps back | chunks |
| hops back | same amount altogether |
| take away | divided by |
| bigger hops | |

Teaching activity

‘We’re going to look at a slow way to work out twenty-four divided by two and a fast way to do it. Show me how you would work out twenty-four divided by two on paper.’

Build on what the child says and does.

If the child doesn’t use a method that can be extended into taking larger chunks, demonstrate a very slow way of working out how many twos make twenty-four.

‘I start with twenty-four and I can take away two at a time like this:

24
– 2
is 22
– 2
is 20
.
.
.

Continue on down the page showing how slow it is to take away two each time.

But be positive about getting to the right answer!

? How many lots of two did we have to take away altogether to get to zero? (*Twelve – let the child count them.*)

? So what is the answer to twenty-four divided by two?

$$24 \div 2 = 12$$

There are 12 twos in 24

? Can you remember that last time we took away bigger chunks at a time? Tell me what you remember about this.

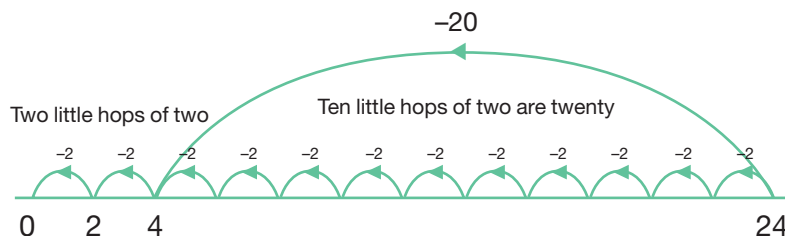
Ask the child to draw a number line and show the slow hops of two back from 24 to 0 (in red pen perhaps).

? Do you remember how easy it is to say the ten times table?

'Look at this 100-square (or multiplication grid) and tell me how many twos there are in twenty.'

Once the child is sure that there are ten twos in twenty, demonstrate (with a green pen perhaps) how to take one enormous step back of ten lots of two.

'One enormous hop of ten lots of two and two more little hops of two make twenty-four.'



'So I can work out how many twos there are in twenty-four by taking twelve little steps, or one enormous hop of ten lots of two, then two more little hops of two.'

Support the child by going over the calculation again.

Clarify that you get the same answer doing it the quicker way because you still count back twelve lots of two. Taking a big hop makes it quicker.

You need to make the link from the number line hops to the earlier vertical recording on paper.

24	
<u>- 20</u>	Ten hops of two
is 4	
<u>- 2</u>	One hop of two
leaves 2	
<u>- 2</u>	One hop of two
0	Twelve hops of two altogether

So $24 \div 2$ is 12.

'We subtracted a bigger chunk doing it this way. Instead of doing little chunks of two each time, we did one big chunk of twenty then two more little chunks.'

? Could we have done it another way?

'We'll do some more of this next time. Once you understand this method of taking larger chunks, it is going to help you to be much quicker at division.'

Spotlight 2

Continues to subtract twos when calculating twenty divided by two without using knowledge that two multiplied by five equals ten

7
Y
4
X
/
÷

Opportunity for: making a problem easier

Go for tens

Time 10–20 minutes

Resources

- Coloured pens
- Number lines
- Bead string
- 100-square and/or *Multiplication grid 1* (Resource sheet 35)

Key vocabulary

- steps back
- chunks
- hops back
- same amount altogether
- take away
- divided by
- bigger hops

Teaching activity

‘We are going to do some counting back today, taking big steps, and this is going to help you to get faster at doing division.’

Show the child a bead string with twenty beads and a number line with 20 marked.

? How could you do twenty divided by two, slowly with lots of little hops along the number line and moving two of these beads at a time?

If the child can’t do this, help them to draw in the steps of two and count along the bead string in twos.

Show the child how their counting back of beads and their hops back of two relate to a long written method.

$$\begin{array}{r} 20 \\ - 2 \\ \hline 18 \\ - 2 \\ \hline 16 \\ - 2 \\ \hline 14 \\ \vdots \end{array} \quad \begin{array}{l} (2 \times 1) \\ (2 \times 1) \\ (2 \times 1) \end{array}$$

Continue this down the page, taking away ten lots of two altogether.

? Do you know what ten lots of two are?

Help the child to refer to a multiplication grid or to a 100-square if necessary.

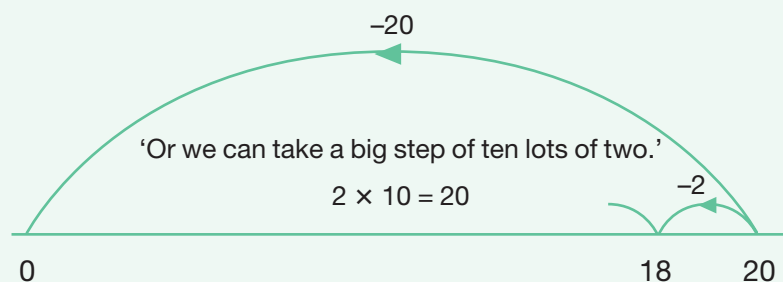
‘Instead of that very big long calculation taking off one two at a time, I could take away ten lots of two all in one big hop back.’

$$\begin{array}{r} 20 \\ - 20 \\ \hline 0 \end{array} \quad (2 \times 10)$$

‘So twenty divided by two is ten. There are ten twos in twenty.’

If the child needs more support to understand this, show the slow counting back in ten hops of two, and the faster counting back of ten lots of two in one big step of twenty.

'We can do ten little steps of two to get from twenty to zero.'



'There are ten twos in twenty, so twenty divided by two is ten.'

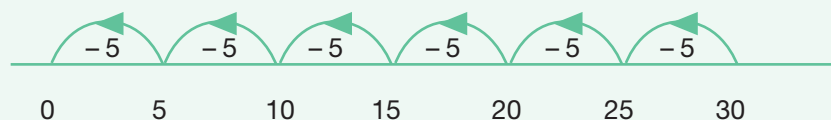
? Work out thirty divided by five on a number line and do it without taking lots of little steps of five.

If the child can't think of how to do that, suggest that they draw a number line and put the numbers on and have a think about it while you write out the slow way of doing the calculation all down a page.

While you write:

$$\begin{array}{r} 30 \\ - 5 \\ \hline 25 \\ - 5 \\ \hline \vdots \end{array} \quad \begin{array}{l} (5 \times 1) \\ (5 \times 1) \end{array}$$

show the child what you are doing and if they cannot think of any bigger hops to make, they could make a little hop of five as you work out the calculation.



? How many hops of five did we need to get from 30 to zero?

? So how many fives are there in thirty? How can we write that down?

$$30 \div 5 = 6$$

'Ten is a very useful number when we are trying to hop back in bigger steps. Look carefully at your six hops of five.'

? Could you take another colour of pen and make bigger steps back?

? What size steps could you make?

Help the child to move towards three hops of ten, or some other large hop.

Go back to the long calculation you did earlier and show how it can be done much more quickly.

$$\begin{array}{r} 30 \\ -5 \\ \hline 25 \\ -5 \\ \hline 20 \\ \vdots \end{array}$$

(5 x 1)
(5 x 1)

This can also be written as:

30		30
subtract 10	(5 x 2)	<u>- 10</u>
leaves 20		20
subtract 10	(5 x 2)	<u>- 10</u>
leaves 10		10
subtract 10	(5 x 2)	<u>- 10</u>
leaves 0	(5 x 6 altogether)	0

So $30 \div 5 = 6$.

- ? How many big steps of ten? (Three)**
- ? How many little steps of five? (Six)**
- ? Why do you think it took half the number of hops when we hopped back in tens?**

If the child is finding it difficult to see the relationship between these hops, go back to the number line hops.

Demonstrate how a hop of five is half the size of a hop of ten.

‘So with hops of ten we only need half the number of hops.’

Spotlight 3

Continues to subtract twos when calculating twenty divided by two without using knowledge that two multiplied by five equals ten

Opportunity for: trying alternative strategies

Bead string fives

Time 15–20 minutes

Resources

- Coloured pens
- Number lines
- Bead string
- *Multiplication grid 1* (Resource sheet 35)
- Place value (arrow) cards

Key vocabulary

- | | |
|-------------|------------------------|
| steps back | chunks |
| hops back | same amount altogether |
| take away | divided by |
| bigger hops | |

Teaching activity

'Today, we are going to use a bead string and a number line to do a division. We are going to work out sixty-six divided by five.'

? If we were going to do this division by taking lots of little steps back from sixty-six, what size would be a good size for the little steps? (Steps of five)

If the child is unsure, show the calculation $66 \div 5$ and ask the child to read it carefully: 'sixty-six divided by five.'

'The size of the steps is five.'

Show on the bead string how you could count back five beads at a time from sixty-six.

? If it was sixty-six divided by seven what would the size of the steps be? (Seven)

? If it was sixty-six divided by eight what would the size of the steps be? (Eight)

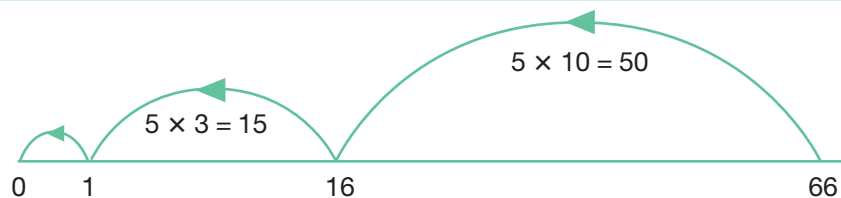
Do a few more examples, if needed.

'For sixty-six divided by five we could do lots of little steps of five, but, instead of that, we are going to try to take at least one very big step first'.

? What big step do you think you could take?

Help the child to think about the five times table, either looking at a multiplication grid or reciting the five times table.

If they don't suggest a suitable first large step, you could suggest a step of ten lots of five or counting back fifty beads on the bead string.



Demonstrate on the bead string and the number line, sliding ten lots of five beads, or one large step of fifty on the number line, three lots of five beads as a smaller step of fifteen and one bead is left over.

If the child seems to be having trouble with splitting sixty-six into fifty and sixteen, you might find it helps to use place value cards.



? How many groups of five have been slid along the bead string? Or can you count them on the number line?

There have been ten lots of five and three lots of five, so sixty-six divided by five is thirteen remainder one.

? Can you write that down starting with the sixty-six and doing a division calculation?

Let the child do this in their own way and you can build on that at a later date if the method is quite slow. Comparing their method with another more efficient method in the class might be the best way to move the child on.

'Can you do sixty-six divided by five in a really efficient way? Whose method is quickest?'



Spotlight 4

Continues to subtract twos when calculating twenty divided by two without using knowledge that two multiplied by five equals ten

Opportunity for: reasoning about numbers

7
Y
4
X
/
÷

Three steps only

Time 15–20 minutes

Resources

- Paper and coloured pens
- Number lines
- Bead string

Key vocabulary

- | | |
|-------------|------------------------|
| steps back | chunks |
| hops back | same amount altogether |
| take away | divided by |
| bigger hops | |

Teaching activity

‘We are going to do some divisions today, and I want you to do them without using more than three steps on the number line.’

? Can you work out forty-two divided by three?

? What could be your first big step?

If the child seems unsure, ask them first how big the little steps would be in that calculation. (The small steps are three.)

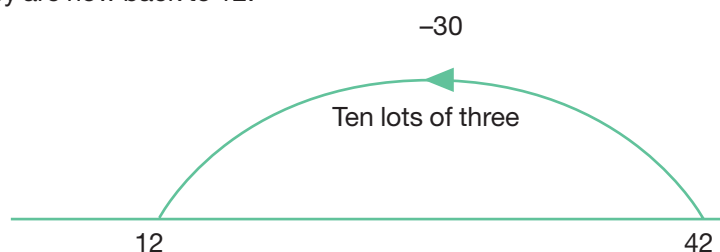
Remind them of how easy it is to take steps ten times bigger.

? What is ten times bigger than three? (Thirty)

‘You can record what you do in any way you want, but you mustn’t take more than three steps back.’

? If you take your first big step of thirty, what number will you get back to?

Make sure they are now back to 12.



‘You must now get to 0 but you only have two steps left.’

Support the child to work out either one more step of four lots of three, or two more steps of two lots of three.

? What is the answer to forty-two divided by three? (Fourteen) How do you know?

If you have time, you could do another example such as seventy-two divided by five. (This one has a remainder.)

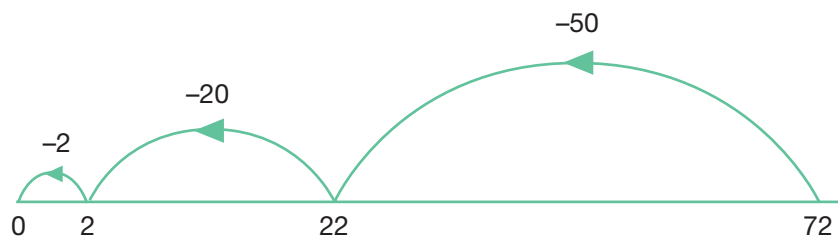
‘Remember you must do it in three steps or fewer.’

? What size steps would you take if you were taking lots of little steps? (*Five*)

? What very big step could you take?

If the child is unsure, remind them again of the multiplication table for tens.

One method might be to do ten lots of five, then four lots of five and there is a remainder of two.



(Some children might feel that they have done this in two steps because the remainder doesn't make a whole step of five.)

? Do you think that you are starting to understand this quicker way of doing divisions? What do you need more help with?

Spotlight 5: a learning check

Continues to subtract twos when calculating twenty divided by two without using knowledge that two multiplied by five equals ten

Opportunity for: discussing and explaining

Chunking chase

Time 10–20 minutes

Resources

- At least two players
- Large folders or books
- Egg-timer or stopwatch
- Rewards

Check: does the child use key vocabulary?

- | | |
|-------------|------------------------|
| steps back | chunks |
| hops back | same amount altogether |
| take away | divided by |
| bigger hops | |

Teaching activity

This game, **Chunking chase**, can be played with children cooperating in pairs racing another pair. Children can hide what they are doing behind a big book propped open on the table.

You can vary the numbers as children get more confident, starting with numbers that fit with taking a first large chunk of ten times the step size and leave no remainder, for example:

$$36 \div 3 \quad \text{or} \quad 48 \div 4 \quad \text{or} \quad 70 \div 5$$

Children can move on to numbers where they might need to take more than one step of ten times the step size, for example:

$$54 \div 2 \quad \text{or} \quad 78 \div 3 \quad \text{or} \quad 92 \div 4$$

Then they can move on to numbers that have a remainder, for example:

$$71 \div 2 \quad \text{or} \quad 83 \div 4 \quad \text{or} \quad 55 \div 2 \quad \text{or} \quad 87 \div 5$$

You can put the division calculations that you want the children to use in a bag, or just give them one calculation at a time for a 5-minute lesson.

How to play

1. Players write down the division calculation.
2. The egg-timer will then be started and the players try to work out the calculation by chunking the numbers so that they can do it quickly.
3. The players record the division as a vertical calculation going down the page or as number line hops.
4. Every player or pair that works it out before the egg-timer runs out will win ten rewards.
5. Then players compare their methods and answers.
6. Players who have worked out the calculation correctly will win a further ten rewards.
7. Write down the score and keep it for next time.

? Could you have done that calculation a different way?

? Which way is best? Why?

Variations

- Play with a stopwatch and write down how many minutes it takes. Players then score that many minutes and this becomes their 'personal best time'. Next time they play the game they have to try to beat it!
- (You need to supervise this and it can take quite a long time to be ready to play.) Each player or pair makes up a division for the other player or pair. But they must know the right answer to any division they give another player! They must write this down first.

Learning outcomes

By the end of this set of activities children should be able to:

- use and understand connected mathematical vocabulary;
- tackle related learning tasks with increased motivation and confidence;
- take steps back along a number line in multiples of the step size;
- record in a vertical way, subtracting multiples of the step size;
- use their knowledge of multiplication to speed up their calculating;
- understand that chunking makes calculating faster.

Misuses half-understood rules about multiplying and dividing by powers of ten and the associative law, for example $145 \times 30 = 145\ 000$

Opportunity for: reasoning about numbers

Resources

- Place value (arrow) cards
- Abacus
- Squared paper
- Calculator

Key vocabulary

- | | |
|-------------------------------|--------------------------|
| multiply by | ten times larger/smaller |
| divide by | two-digit numbers |
| digits move to the left/right | three-digit numbers |

Teaching activity

Time 15–20 minutes

‘We are going to look at multiplying and dividing by ten, a hundred and a thousand, and we are going to use a calculator so that we can see some patterns.’

Put a single-digit number, for example 5, into the calculator and show it to the child.

? Can you tell me how you could change the screen to 50 in one move using the multiplication key?

? Can you explain what you did?

Make sure the child uses the correct vocabulary.

If the child is unsure, show how to key in $\times$ 1 0 = to show 50, and use the correct vocabulary to explain it.

‘I had five and I multiplied by ten and that makes fifty. Fifty is ten times larger than five because we multiplied it by ten.’

? Can you predict what will happen if we key in seven multiplied by ten on the calculator?

Show the numbers with place value cards.

$$\begin{array}{|c|} \hline 7 \\ \hline \end{array} \times 10 = \begin{array}{|c|c|} \hline 7 & 0 \\ \hline \end{array}$$

Ensure that the child uses appropriate language to explain the process with seven. For example, ‘seven multiplied by ten is seventy’ and ‘seventy is ten times larger than seven’.

Go back to 5 on the calculator.

? What happens when we multiply a single-digit number by a hundred?

Move on to multiplying by a thousand, and to multiplying two-digit numbers by ten, a hundred and a thousand, continuing to ask the child to explain their choices of calculator buttons using appropriate vocabulary.

Ask the child to talk you through five multiplied by a hundred with place value cards, and maybe with an abacus.

If the child tends to use language such as ‘you add a nought’, you will need to introduce a more accurate way of explaining the changes in the numbers. For example, ‘The five moves to the left and becomes ten times larger, so we put a zero in the empty column to fill the empty space.’

It can be helpful to draw up some charts on squared paper and to name the tops of the columns: units or ones, tens, hundreds, thousands, and so on.

Show how a number changes, using the calculator, place value cards or abacus, or whatever the child chooses to use.

Th	H	T	U
			5
		5	0
	5	0	0
5	0	0	0

$$\begin{aligned} \times 10 &= 50 \\ \times 10 &= 500 \\ \times 10 &= 5000 \end{aligned}$$

? Is this true: $5 \times 1000 = 50 \times 100$?

? How do you know?

? Is this true: $125 \times 20 = 12\,500$?

? How do you know?

Support the child to move towards explaining:

$$125 \times 20 \text{ is } 125 \times 10 \times 2 = 1250 \times 2 = 2500$$

Recognising and naming larger numbers needs to be secure to understand and communicate this aspect, so you might need to spend time using the calculator to explore hundreds and thousands, encouraging the use of appropriate vocabulary to explain.

? Can you key in sixty-five and tell me what number is ten times bigger? What do you need to do to key in ten times bigger? Now multiply by ten and see if you were right.

? What if I put twenty-six into the calculator – can you change it to two thousand six hundred with one multiplication? How would you write that down?

Record some of the patterns emerging (squared paper can help).

Multiplying 48 by ten

HTh	TTh	Th	H	T	U
				4	8
			4	8	0
		4	8	0	0
	4	8	0	0	0

$$\begin{aligned} \times 10 &= 480 \\ \times 10 &= 4\,800 \\ \times 10 &= 48\,000 \end{aligned}$$

Do you know the names of the columns?

U = units or ones

T = tens

H =

Th =

TTh =

? Tell me about a pattern you can see.

? Can you name this number: 480 000? (four hundred and eighty thousand)

Help the child to talk about what they can see happening and help them by recording some generalisations in their own words. For example:

Every time we multiply by ten, the digits move one place to the left.

If we multiply a single-digit number by a thousand (that has three zeros) the single-digit number moves three places to the left into the thousands column and the hundreds, tens and ones columns all have zeros to fill the empty spaces.

? What do you think will happen when we divide by ten?

? What if I key in three hundred and sixty – what single division will get the screen to thirty-six?

Let the child explore using a calculator, and record some of their findings, explaining the process which they are using and any patterns they notice.

Dividing 960 000 by ten

HTh	TTh	Th	H	T	U
9	6	0	0	0	0

$$\div 10 = 96\,000$$

? What shall we write down so that we remember it for next time?

Spotlight 1

Misuses half-understood rules about multiplying and dividing by powers of ten and the associative law, for example $145 \times 30 = 145\ 000$

Opportunity for: exploring multiplication patterns

Factor trees

Time 15–20 minutes

Resources

- Squared paper
- Calculator
- *Multiplication grid 1 or 2* (Resource sheet 35 or 36)

Key vocabulary

multiply by	two-digit numbers
divide by	three-digit numbers
digits move to the left/right	factors
ten times larger/smaller	

Teaching activity

‘We are going to find some factors of numbers to make it easier to multiply.’

? Is this correct: $8 \times 20 = 800$?

? How do you know?

Support the child to move towards:

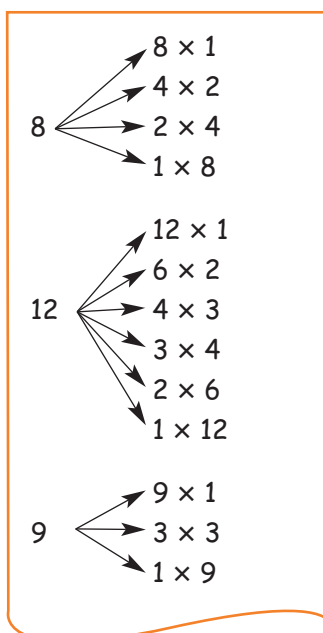
$$8 \times 20 = 8 \times 10 \times 2 = 80 \times 2 \text{ or } 8 \times 2 \times 10 = 16 \times 10.$$

Support the child to see that you can use factors of 20 to help you to multiply by 20.

Draw a few factor trees, emphasising that one, eight, two and four are factors of eight, enlisting the child’s help in identifying factor pairs, using the multiplication grid for support.

If the child finds that difficult, use cubes to make arrays. For example, with eight cubes we can make an array of one row of eight, or two rows of four.

Keep these recordings for later use.



‘Some numbers have many more factors than others.’

Record some multiplication calculations using factors. Keep these recordings for use in later sessions.

? Use factors to help you to work out eighteen multiplied by twelve.

Let the child use a calculator to work out several examples, so that they have experience of using different factors. Encourage them to talk you through what they are doing as they do it, and to comment on anything they notice.

They could break down eighteen multiplied by twelve into factors of eighteen as well as twelve, resulting in: $2 \times 9 \times 6 \times 2$.

Whichever order you multiplied these numbers in, you would still get the same answer.

? Tell me what we have been doing to try to make numbers easier to multiply.

? Give me an example.

? Why does using factors help?



? Make a factor tree for twenty-four. Now multiply twenty-seven by twenty-four using factors.



If no one uses the nearness of twenty-seven to twenty-five to help them in this calculation, point out that they could use this as a check.

Spotlight 2

Misuses half-understood rules about multiplying and dividing by powers of ten and the associative law, for example $145 \times 30 = 145\ 000$

Opportunity for: discussing and making decisions

.....

Oops! banana slips

Time 15–20 minutes

Resources

- Cubes or counters

Key vocabulary

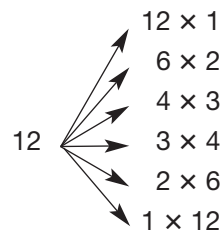
- | | |
|-------------------------------|------------------------------|
| multiply by | three-digit numbers |
| divide by | partition into tens and ones |
| digits move to the left/right | array |
| ten times larger/smaller | factors |
| two-digit numbers | |

Teaching activity

‘We are going to do some more work with factors to help us to multiply. There is more than one way of multiplying, and we need to be very clear about what we are doing so we don’t slip on a banana skin!’

? Tell me the factors of twelve and draw a twelve factor tree.

Support the child to draw:



'We're going to work out seven multiplied by twelve using these cubes and a pair of factors of twelve. Let's start by using factors three and four and I'll record for you what we do.'

7×12 can be written as $7 \times 3 \times 4$.

? Make an array for seven multiplied by three with cubes.

This array shows 7×3 .

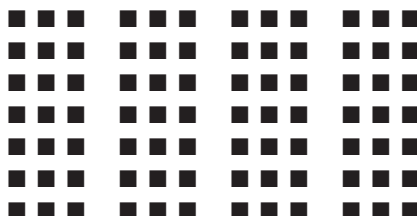


? Look carefully at $7 \times 3 \times 4$ and tell me what we need to do to our array of 7×3 to make it show 7×12 .

Support the child, moving towards the idea that we need four lots of the 7×3 cubes to show 7×12 .

This array shows four lots of 7×3 cubes:

$$7 \times 3 \times 4$$



$$7 \times 3 = 21 \text{ and } 21 \times 4 = 84.$$

? Rearrange these same cubes to give another way to show 7×12 .

The child might need support here to recognise that they need to choose another pair of factors of twelve, for example six and two, then arrange the same eighty-four cubes into different arrays, for example an array of seven multiplied by six twice.

? What did you learn today about the way factors can help us to multiply?



? How would you explain to someone why multiplying eighteen by twelve gives the same answer as $2 \times 9 \times 2 \times 6$?

Spotlight 3

Misuses half-understood rules about multiplying and dividing by powers of ten and the associative law, for example $145 \times 30 = 145\ 000$

Opportunity for: exploring place value and multiplication patterns

1 Y6 $\times$ / $\div$

Multiply it

Time 15–20 minutes

Resources

- Place value (arrow) cards
- Wipe-clean place value board
- Calculator

Key vocabulary

- | | |
|-------------------------------|--------------------------|
| multiply by | ten times larger/smaller |
| divide by | two-digit numbers |
| digits move to the left/right | three-digit numbers |

Teaching activity

‘We’re going to do some more multiplying by ten and a hundred today. Then we will multiply by twenty and two hundred using factors.’

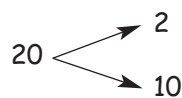
? What can you remember about multiplying by ten and a hundred?

? What is three multiplied by ten and three multiplied by a hundred?

If the child has forgotten, go over multiplying by ten and a hundred using a calculator and place value cards, recording on a place value board. Try to work out which model the child seems to prefer.

Record some multiply by ten and multiply by a hundred calculations. Then extend these to some related multiply by twenty and multiply by two hundred calculations, guiding the child towards the patterns using appropriate vocabulary to support description of the process.

$3 \times 10 = 30$ and $3 \times 20 = 60$ as this is $3 \times 10 \times 2$
because factors of twenty are ten and two.



$3 \times 100 = 300$ and $3 \times 200 = 600$ as this is $3 \times 100 \times 2$.

If the child needs it, go through this pattern again with four and five. Use a calculator so that the child can focus on the patterns, rather than worrying about getting the calculation right. Record the patterns of numbers. Encourage the child to explain in their own words what they notice.

Again, a place value board and/or place value cards might assist in building up appropriate images.

$5 \times 10 = 50$ and $5 \times 20 = 100$ as this is $5 \times 10 \times 2$

$5 \times 100 = 500$ and $5 \times 200 = 1000$ as this is $5 \times 100 \times 2$

? Tell me how to use the ten times table to multiply six by eighty.

Support the child in finding a way of doing this, for example $6 \times 10 \times 8$.

Encourage the child to work out all the versions they have managed to identify for six multiplied by eighty.

Repeat with another example.

? What shall we write down today that is important to remember for next time?

Spotlight 4

Misuses half-understood rules about multiplying and dividing by powers of ten and the associative law, for example $145 \times 30 = 145\ 000$

Opportunity for: exploring patterns with division

1 Y6 $\times$ / $\div$

Division patterns

Time 15–20 minutes

Resources

- Place value (arrow) cards
- Wipe-clean place value board
- Number cards
- Squared paper
- Calculator

Key vocabulary

- multiply by
- divide by
- digits move to the left/right
- ten times larger/smaller
- two-digit numbers
- three-digit numbers

Teaching activity

‘We are going to make some division patterns today, dividing by one, by ten and by a hundred.’

? Can you show me the division key on your calculator?

? What can you tell me about these number patterns?

$$\begin{aligned} 3 \div 1 &= 3 \\ 30 \div 10 &= 3 \\ 300 \div 100 &= 3 \end{aligned}$$

? Can you make another pattern with three calculations involving division by one, ten and a hundred, all with the answer 4 on your calculator?

? Explain your choice of numbers.

Build on what the child does.

If the child needs a bit more guidance, you could start off a pattern and, if they still seem not to be following, use place value cards or a place value board and number/arrow cards, or whatever the child seems to prefer.

Th	H	T	U	
3	0	0	0	$\div 10 = 300$
	3	0	0	$\div 10 = 30$
				$\div 10$
				$\div 10$

? Tell me about your pattern.

Encourage the child to make generalised statements such as:

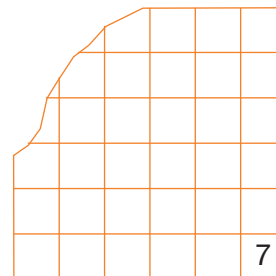
‘When you divide by ten the digits move one place to the right.’

? What do you think would happen if we divided by a hundred?

If the child can’t see that the digits would move two places to the right each time, explore this on a calculator with them, using any supporting equipment which the child seems to prefer to use. Encourage the use of appropriate vocabulary.

Move on to using another squared paper grid.

‘Show me a division by ten pattern that ends in seven.’ (Ending in forty-seven would be more challenging.)



? Was there anything we worked on today that you thought was new knowledge for you?



? I divided my mystery number by a hundred and did that division by a hundred four times and ended with eight. What was my mystery number? (Eight hundred million, 800 000 000)

(Note: The division is by one hundred four times, not by four hundred! You could use this as a moment to point out that we have to be very careful about the language we use in mathematics.)

Spotlight 5: a learning check

Misuses half-understood rules about multiplying and dividing by powers of ten and the associative law, for example $145 \times 30 = 145\ 000$

Opportunity for: discussing and explaining

.....

Beat the calculator **Time** 5–15 minutes

Resources

- Wipe-clean place value board
- Calculators
- A group of four or more children (a whole class is ideal)

Check: does the child use key vocabulary?

multiply by	two-digit numbers
divide by	three-digit numbers
digits move to the left/right	factors
ten times larger/smaller	

Teaching activity

‘Today we are going to play a game to help you practise multiplying and dividing by multiples of ten.’

This game, **Beat the calculator**, is a small- or large-group game and works well with the whole class. The game can be played at many different levels and can be used to give practice and consolidation of a wide range of learning objectives.

How to play

1. Divide the children into two groups. Give the children in one group a calculator each but give nothing to the second group. Tell the second group that they can make jottings on paper if they want to.
2. Read out a series of calculations. Here are a few examples:

 45×100 100×2 $30 \div 10$ 29×100 $450 \div 10$
3. The group with calculators must key each calculation in, and not call anything out until they see the answer on their screen.
4. The group with no calculators must work out the calculation in their heads, or with brief jottings, and then call out the answer.

(Depending on the calculations which you read out, the group without the calculators could beat the children with calculators! That is a very valuable lesson, making this game one to play at least once a term.)

? Why could you work that out so quickly in your head? Can you tell us how you did it? Did anyone do it a different way?

5. Give a hundred points to the group that shouts out the answer first and keep the scores on the board.
6. After about five or ten goes, swap the groups round and call out five or ten more calculations.

Variations

- Let one of the children work out about ten questions of their own which they think could be worked out more quickly mentally than on the calculator. That child is then the leader and calls out their questions.

**? Can you explain to me how you worked that out?
Is there another way to do it?**

- This can be a good game to share with parents, especially those who think that calculators harm children's mathematics!

Learning outcomes

By the end of this set of activities children should be able to:

- use and understand connected mathematical vocabulary;
- tackle related learning tasks with increased motivation and confidence;
- multiply and divide by ten and a hundred;
- describe what is happening to the digits in multiplying and dividing by powers of ten;
- multiply by multiples of ten, a hundred and a thousand;
- use factors to support multiplication;
- use a calculator appropriately.

Has difficulty, when appropriate, interpreting a remainder as a fraction, for example $16 \div 3 = 5\frac{1}{3}$

Opportunity for: communicating with a range of mathematical language and symbols

Resources

- Cubes/apples/rods/counters
- Calculator
- Empty number lines
- Plastic, paper or dough fraction pieces or *Fractions* (Resource sheet 42)

Key vocabulary

remainder	whole one
left over	half
fraction	quarter
divided by	

Teaching activity

Time 10–20 minutes

Explain to the child that today's activity will help them to be clearer about how to deal with remainders when they work out divisions.

? What do you think it means to have a remainder after working out a division calculation?

'Choose something on the table to show me what you mean.'

Listen to what the child says and make a note of what they do.

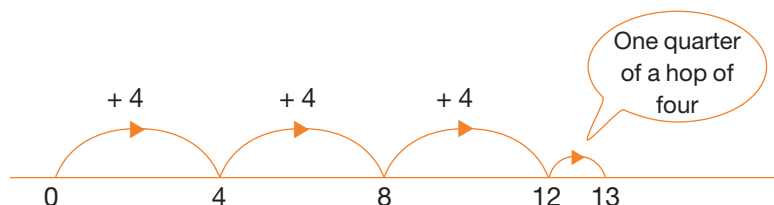
If the child seems to be unsure about remainders, give them a simple calculation to work out.

? Can you work out thirteen divided by four? You can use any of the equipment to help you.

It is likely that some children will work out thirteen divided by four by using a sharing method – putting thirteen counters into four groups. Remind the child that when we say 'divided by four' we mean putting the items into fours, not shared between four.

Let them work out how to show the calculation with cubes/dough/apples, and remind them how to use a number line to work out the calculation. (Hops backward or forward are both fine – the child should choose whichever representation they prefer.)

'There are three hops of four to get to 12 and then a quarter of a hop to get to 13.'

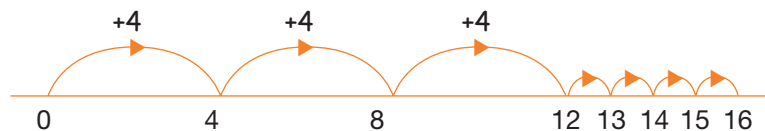


If the child seems to be unsure about naming fraction parts, you might need to spend a few moments identifying halves, quarters and three-quarters using paper or plastic fraction parts.

Record with the child, encouraging them to use a range of appropriate language and using pictures and symbols that help them. (The number line picture shows remainders as fractions in a powerful way, so focus on that.)

$$13 \div 4 = 3 \text{ remainder } 1$$

That remainder is one quarter of the way towards 16, which is the next hop of four.



So $13 \div 4 =$ three groups of four and a quarter of a group of four.

$13 \div 4 = 3\frac{1}{4}$ because the remainder of one is a quarter of a whole hop.

Try another example and record it with the child.

? What is fifteen divided by two?

? What picture do you have in your head for that?

Talk the child through the images, putting fifteen into equal groups of two, steps of two on a number line. Dividing by two means 'how many twos in fifteen?'

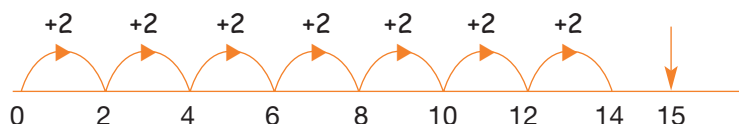
$$15 \div 2 = 7 \text{ remainder } 1$$

You can say this as: how many twos make fifteen? It is seven with one left over.

Fifteen divided by two makes seven groups of two and one left over.

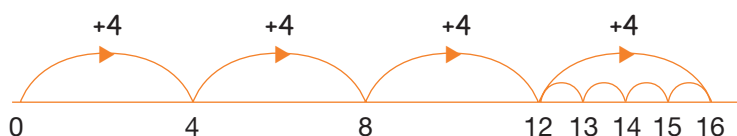
Fifteen buns put into packs of two will make seven packs and one bun left over (which is half a pack).

We can do seven hops of two and a half hop of two (one). A whole hop of two at the end would get us to 16, but we only want to go halfway (to 15).



'Let's draw thirteen divided by four on a number line and look very carefully at that last hop.'

'Thirteen divided by four means we need to take hops of four.'



'We can make three complete hops of four and the remainder is a quarter of a whole hop of four.'

'The whole hop of four would get us to 16, but we only want to get to 13. So thirteen divided by four is three and a quarter groups.'

? If you key in thirteen divided by four on your calculator, what do you think the answer would be? Explain how you know that.

If the child is unsure about how fractions look on a calculator, you might choose to draw up a table of fractions and how they relate to decimals.

The child can key these in as $1 \div 2$, $1 \div 4$, and so on.

$1 \div 2 = 0.5$, which is a half ($\frac{1}{2}$).

$1 \div 4 = 0.25$, which is a quarter ($\frac{1}{4}$).

$1 \div 3 = 0.3333$, which is a third ($\frac{1}{3}$).

$3 \div 4 = 0.75$, which is three-quarters ($\frac{3}{4}$).

? What shall we record today that is important to remember for next time?

Keep recordings of any pictures and symbols which the child uses, ready for later work.

Spotlight 1

Has difficulty, when appropriate, interpreting a remainder as a fraction, for example $16 \div 3 = 5\frac{1}{3}$

Opportunity for: solving real-life problems

Apples and buns

Time 15–20 minutes

Resources

- Dough and blunt knife, apples, carrots, buns
- Plastic or paper fraction parts or *Fractions* (Resource sheet 42)
- Calculator

Key vocabulary

remainder	half
left over	quarter
fraction	three-quarters
divided by	thirds
whole one	fifths

Teaching activity

‘We are going to do some more division today. We are going to find answers where we interpret the remainder as a fraction.’

Using sixteen apples, or other objects, ask the following:

- ? We have sixteen apples for three horses. How many apples is that each? What about the one left over?**



‘Each horse can have five apples and there is one left over, so $16 \div 3 = 5$ remainder 1.’

- ? How can we divide up the one apple left over?**

Support the child to show how the apple can be cut into three equal parts.

- ? What name do we call each of these parts?**

If the child can't name the fraction as a third, spend a bit of time naming fraction parts. Check that they know half, quarter, three-quarters, third and fifth.

Using nine carrots, or other objects, ask the following:

- ? We have nine carrots for two rabbits. How many carrots is that each?**

$9 \div 2 = 4$ remainder 1, but if we divide the left-over carrot by two, we can write this as:

$$9 \div 2 = 4\frac{1}{2}$$

The calculator showed an answer of 4.5.

Using seventeen buns, or other objects, ask:

- ? We have seventeen buns for four children. How many buns is that each?**

$$17 \div 4 = 4 \text{ remainder } 1$$

$$17 \div 4 = 4\frac{1}{4}$$

The calculator showed an answer of 4.25.

- ? Look back at what we have done today and show me a division calculation where the answer includes a fraction.**

Spotlight 2

Has difficulty, when appropriate, interpreting a remainder as a fraction, for example $16 \div 3 = 5\frac{1}{3}$

Opportunity for: identifying patterns

Looking for patterns

Time 15–20 minutes

Resources

- Multiplication grid 1 or 2 (Resource sheet 35 or 36)
- Paper or plastic fraction pieces or *Fractions* (Resource sheet 42)
- Number line
- Counters

Key vocabulary

remainder	whole one
left over	half
fraction	quarter
divided by	three-quarters

Teaching activity

‘We are going to divide by four and find some remainders, and then work out those remainders as fractions.’

? Can you remember from the last session which fractions we might make when we are dividing by four?

Record a list of divisions, asking them:

? What do you think comes next in the pattern?

Use the language ‘nine divided by four is two remainder one’, and then explain that the remainder of one can be divided by four to make four quarters. So the answer to nine divided by four is two and a quarter.

$$\begin{aligned}9 \div 4 &= \\10 \div 4 &= \\11 \div 4 &= \\12 \div 4 &= \\13 \div 4 &= \\14 \div 4 &= \end{aligned}$$

? Can you predict which of these division calculations won’t have a remainder? How do you know?

If the child is unsure about this, encourage them to refer to the four times table on their *Multiplication grid* (Resource sheet 35 or 36).

If the child is not following this, you might want to start the pattern on another bit of paper with $4 \div 4 = 1$ with no remainder.

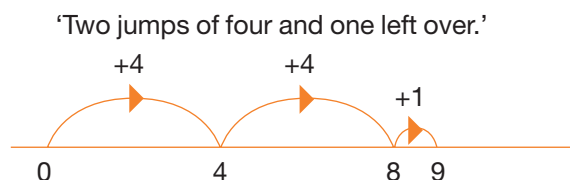
$5 \div 4$ has a remainder of 1, so $5 \div 4 = 1$ remainder 1, which is $1\frac{1}{4}$.

$6 \div 4 = 1\frac{2}{4}$ or $1\frac{1}{2}$

$7 \div 4 = 1\frac{3}{4}$ and so on.

Try to establish that all the calculations that can be related to the four times table have no remainders because these numbers (4, 8, 12, 16, and so on) are multiples of four.

You might choose to model the calculations on both a number line and using counters put into fours.



‘Nine divided by four is two groups of four, and one left over.’



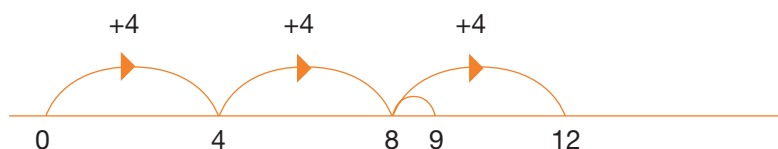
‘If we divided that one left over by four, we get a quarter.’

‘Nine divided by four is two groups of four and one quarter of a group of four.’

? What patterns can you see in the fraction parts of the answers?

$9 \div 4 = 2 \text{ r } 1 \text{ or } 2\frac{1}{4}$
 $10 \div 4 = 2 \text{ r } 2 \text{ or } 2\frac{2}{4} \text{ or } 2\frac{1}{2}$
 $11 \div 4 = 2 \text{ r } 3 \text{ or } 2\frac{3}{4}$
 $12 \div 4 = 3$
 $13 \div 4 = 3 \text{ r } 1 \text{ or } 3\frac{1}{4}$
 $14 \div 4 = 3 \text{ r } 2 \text{ or } 3\frac{2}{4} \text{ or } 3\frac{1}{2}$
 etc.

You could show this on number lines. Each time one more quarter step is added.



‘Nine divided by four is two jumps of four and one quarter jump.’

Ten divided by four is two jumps of four and two quarter jumps.

Eleven divided by four is two jumps of four and three quarter jumps.

Twelve divided by four is three whole jumps of four.’

? Predict the next calculation in the pattern.

? What fraction will the remainder make? How do you know?

? Did you learn anything new today about remainders when we work out division calculations?



? Write in words two division calculations that have the words ‘divided by’ and that have a fraction as part of the answer.

? Write a division calculation that has one-fifth as part of the answer.

Spotlight 3

Has difficulty, when appropriate, interpreting a remainder as a fraction, for example $16 \div 3 = 5\frac{1}{3}$

Opportunity for: reasoning about numbers

Sensible answers

Time 15–20 minutes

Resources

- 28 eggs (or egg substitutes, such as ping pong balls)
- 5 egg boxes
- Calculator
- Money

Key vocabulary

remainder	quarter
left over	three-quarters
fraction	thirds
divided by	fifths
whole one	sixths
half	

Teaching activity

‘Sometimes when we are working out division calculations with real things, we have to be very careful about how we write the answer, making sure it is a sensible answer. We’re going to work on this today.’

? How could you divide twenty-eight eggs by six?

Let the child work this out any way they want.

Note: If the child is trying to share the twenty-eight into six groups, they need to be reminded that the calculation is to *divide by six*, so that means making equal groups of six.

? How would you write down twenty-eight divided by six?

$$28 \div 6 = 4 \text{ remainder } 4$$

If the child is finding it difficult, let them start with twenty-eight eggs and put them into groups of six (in egg boxes if available).

Six multiplied by four is twenty-four and there will be four eggs left over, so one egg box will not be full but can hold the remainder of four eggs.

? Do you think it might help to check it on a calculator? (This gives an answer of 4.6666666.)

? What is the sensible answer to twenty-eight eggs divided by six? How do you know?

Establish that working out the calculation so that the answer has a fraction doesn’t always give us a sensible answer.

? Three children have been doing some odd jobs around the house, and for their neighbours, and, altogether, they have earned £13 which they want to divide equally between themselves. How much will they get each?

If the child needs support, let them work it out using coins or jottings.

£13 divided into three equal groups can be worked out as:

$3 \times 4 = 12$ so each child would get £4, with £1 left to divide up.

100p divided by three is 33p and a bit left over.

? Do you think the calculator might give us a better answer?

$13 \div 3 = 4.33333$

Again, stress that calculator answers don't always make sensible answers.

Talk about what the children might do once they all have £4.33.

? Is it possible to divide the left-over one penny between three people?

? How might they solve their problem? *(Accept any sensible suggestion – they might give the penny to the charity box in the corner shop, or they might have only £4.30 each and give the remaining ten pence to charity or to a younger child who was too small to help them.)*

? For indoor hockey, the thirty-two children in the class need to get into teams of ten.

? What division do you need to do to find out the number of teams?

Help the child to write thirty-two into teams of ten as $32 \div 10 = 3$ teams of ten and two children left over.

'Let's see what the calculator says for thirty-two divided by ten.'

? If you think very hard, can you predict the answer to thirty-two divided by ten?

The calculator gives an answer of 3.2, but 0.2 of a team doesn't have much meaning!

? How could the thirty-two children solve their problem with the two extra children? *(Accept a wide range of suggestions.)*

? What do you have to do to make sure you have chosen a sensible answer when you are giving answers with remainders or fractions?

Spotlight 4

Has difficulty, when appropriate, interpreting a remainder as a fraction, for example $16 \div 3 = 5\frac{1}{3}$

Opportunity for: developing mental images

2 Y6 $\times$ $\div$

Counting backwards or forwards?

Time 10–20 minutes

Resources

- Soft modelling material

Key vocabulary

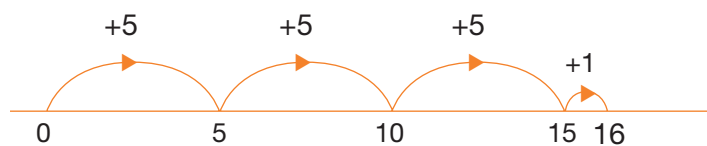
remainder	quarter
left over	three-quarters
fraction	thirds
divided by	fifths
half	

Teaching activity

‘We are going to do some divisions by working them out on an empty number line.’

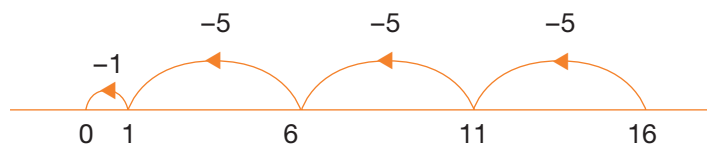
- ?** Can you show me how you could work out sixteen divided by five, drawing yourself an empty number line?

If the child struggles with this, help them to draw a line and draw steps of five. They can start at 0 and count forwards in fives, or start at 16 and count back in fives. Either way they should have a remainder of one, which can be interpreted as one-fifth.



‘Three steps of five and a step of one.’

‘Sixteen divided by five is three groups of five and a remainder of one divided by five is one-fifth.’



- ?** Which way do you like to do your dividing on a number line? Do you like counting forwards, or do you prefer counting backwards?

Try both ways. What do you notice?

$$16 - 5 - 5 - 5 = 1$$

- ?** What shall we do with the one left over? How will we divide it into five equal groups?

- ?** What is the answer to sixteen divided by five? ($3\frac{1}{5}$ or 3.2)

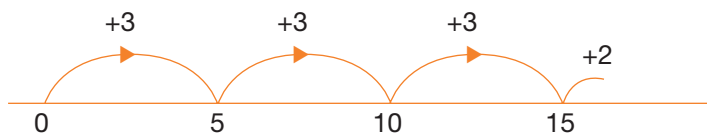
? Work out seventeen divided by three.

'Seventeen divided by three. That is five steps of three along the number line and a remainder of two.'

$$17 \div 3 = 5 \text{ remainder } 2$$

The remainder two when divided by three gives two-thirds ($\frac{2}{3}$).

Because it is divided by three, each step is a step of three, except for the remainder, which is two bits out of three.



'I can count forwards in steps of three.'

The child might find the fraction 'two-thirds' too demanding, so you might need to fold paper into thirds to demonstrate two-thirds. (Folding a piece of A4 paper into thirds to put in an envelope is a skill that some adults find difficult!)

Ask the child to work out nineteen divided by four.

If the child is finding this too challenging, you might want to repeat this lesson with different numbers next time and try to put emphasis on doing the steps just in the direction the child seems to prefer. Many children like to start at 0 because counting forwards uses numbers that are familiar to them and they can find them on their multiplication grid.

'Let's record some of the words and ideas we use for dividing.'

Using multiplication tables helps me.

How many threes make 18?

$$18 \div 3$$

Eighteen divided into three equal groups.

$$9 \div 2$$

will have a remainder

I know all
this about
dividing

I can hop forward on a number
line to divide by a number:

$$5 + 5 + 5 + 1$$

$$16 - 5 - 5 - 5 = 1$$

Remainders can
be interpreted
as fractions.

Spotlight 5: a learning check

Has difficulty, when appropriate, interpreting a remainder as a fraction, for example $16 \div 3 = 5\frac{1}{3}$

Opportunity for: explaining and discussing

Double-decker pizza

Time 5–20 minutes

Resources

- Number cards 1–15 or to suit your children (Resource sheets 1 and 2)
- $\div$ and $=$ cards (Resource sheet 8)
- Plastic fraction pieces or *Fractions* (Resource sheet 42)
- At least one other child

Check: does the child use key vocabulary?

remainder	half
left over	quarter
fraction	three-quarters
divided by	thirds
whole one	two-thirds

Teaching activity

‘This game, **Double-decker pizza**, will help you to get better at making remainders into fractions when you work out division calculations. In this game you must remember to choose two numbers that will give you a remainder, otherwise you won’t be able to have slices to make up your pizza!’

Choose number cards to suit your children, for example 2, 3 and 4, with cards 8–15.

Put the cards on the table face up in two groups: the 2, 3 and 4 together, and 8–15 on the other side of the table.

Put the fraction pieces on the table, face up. If you use the resource sheet you will need to make several copies of it and cut up the fraction pieces.

The game can be played cooperatively, with pairs trying to make two pizzas.

How to play

1. The players take turns to pick up two cards, one card from 2–4 to show how many to divide by (or the size of step on the number line) and the other from the 8–15 cards to show the number to put on the right-hand end of the number line.



2. The player reads out their number sentence: ‘Fourteen divided by three.’
3. Everyone then works out the calculation, for example on their own empty number line.
4. The player then says their answer – in this case, four remainder two – and makes the remainder into a fraction – in this case two-thirds.
5. If everyone agrees that they are right, the player will win a two-thirds fraction piece.
6. The cards are put back on the table and the other player chooses two more cards – at least one of which must be different from the first player’s. So if a player has just divided by three, the new player must divide by either two or four.

7. If children are competing, the winner is the first player to make a whole pizza (for a 5-minute game with two players). For a longer game, the winner needs to make two pizzas to make a double-decker.

- ? Which fraction do you need to make to complete your pizza? So which number cards would be good to choose?**
- ? Can you explain to me how you knew to choose those two numbers to complete your pizza?**
- ? Could you have chosen different cards to get the same fraction answer?**
- ? Is it always possible to make a whole pizza in two goes?**
- ? How do you know?**

Variations

- Play with different number cards, for example a complete set of 1–20 or 30. For this game you need to have lots of paper circles and make your own fraction piece of pizza if it isn't on the resource sheet or in your plastic fraction set. (For example, if you divide by five you might need several fifths of circles.)
- Play with just a small selection of cards but have them face down (but still in the two sets). (This involves more luck but gives less opportunity to use reasoning to choose numbers to complete a pizza. This might give a child who is having more difficulty with the game a greater chance of winning. Allow at least 15 minutes for this game.)



- ? Why can you never make a whole pizza in one go with these cards?**
- ? Which number cards would you need to make a whole pizza in one go? (With two sets of cards so you could make, for example, $2 \div 2 = 1$ so you wouldn't be making a fraction.)**
- ? Using any numbers up to a hundred, write five calculations that would give the same size fraction part in the answer.**

Learning outcomes

By the end of this set of activities, children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- interpret a remainder as a fraction;
- interpret 'divide by' as a grouping calculation;
- work out a grouping calculation on a number line and show and name the fraction part.

Interprets division as sharing but not as grouping (repeated subtraction) so is unable to interpret a calculation such as $12 \div \frac{1}{2}$

Opportunity for: solving real-life problems

Resources

- Apples, muffins, chocolate bars, dough
- Plastic fraction parts or *Fractions* (Resource sheet 42)

Key vocabulary

halves	divided by
quarters	how many halves in ...?
whole ones	divided into halves
fractions	

Teaching activity

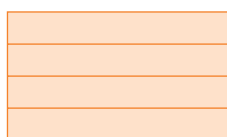
Time 15–20 minutes

‘We are going to do some dividing by fractions. I’ve got some apples/muffins/chocolate/dough to help you understand what we mean by dividing by fractions.’

‘I’m going to write down some of the words and symbols that we use when we divide by fractions.’

Show a chocolate bar (or paper copy) that could be divided into quarters.

? How many quarters in a whole one?



? How many quarters can you get from three whole ones?

If the child is unsure about the word ‘quarter’, you will need to do a variety of practical activities focusing on fractions. Understanding fractional parts needs to be secure before working through this set of activities.

You or the child can draw the three chocolate bars to show the twelve quarters and record words and symbols.

Words and symbols we use to divide by fractions.



How many quarters in three whole ones?

Three divided by a quarter is twelve.

Three divided into quarters is twelve.

$$3 \div \frac{1}{4} = 12$$

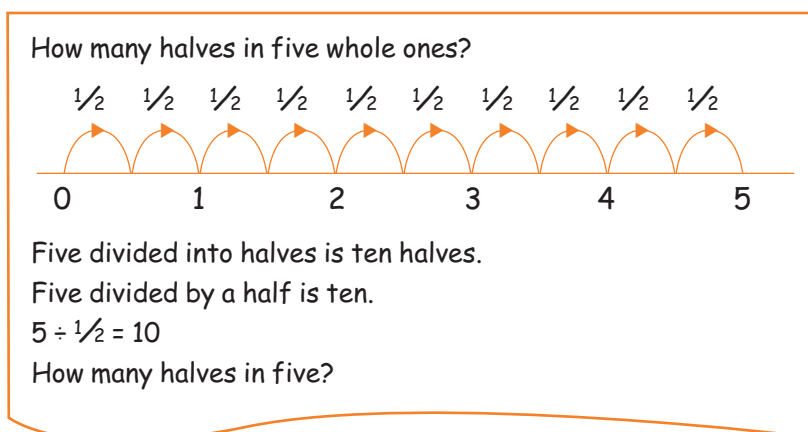
? How many half apples can you get from five whole ones?

Let the child work this out any way they choose.

If the child can't seem to visualise what is needed, use apples or paper strips to improve accuracy and actually cut the five apples into halves, or fold and cut the paper.

You might want them to draw what they did on the recording chart.

It would be helpful to talk through and draw at least one calculation on a number line.



**? Can you use these six muffins to make up a division sentence with 'divided by half' in it?
How did you work that out?**

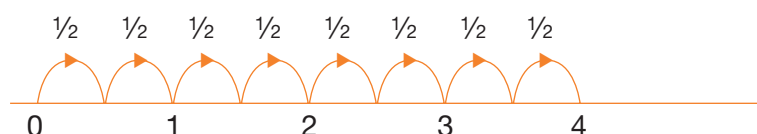
If the child needs more help doing this, let them work practically with further objects, such as dough, paper strips, and so on.

Give more examples and encourage them to make up their own ones, such as 'Fourteen oranges divided by a half means that twenty-eight chimps can have a half each.'

You could do a recording in which they select the right words from your previous recordings.

So on a number line you could draw six divided by a half, showing the twelve half hops that can be made.

? Can you make up a 'dividing by' number sentence that will go with this number line picture?



This could be 'Four apples divided by a half makes eight halves.'

? What did you learn today about dividing by fractions?

Keep all the recordings of words, symbols and pictures for next time.

Spotlight 1

Interprets division as sharing but not as grouping (repeated subtraction) so is unable to interpret a calculation such as $12 \div \frac{1}{2}$

Opportunity for: developing mathematical vocabulary

3 Y6 x / ÷

Dividing words

Time 15–20 minutes

Resources

- Apples, muffins, chocolate bars, dough, blunt knife
- Plastic fraction parts or *Fractions* (Resource sheet 42)

Key vocabulary

halves	how many halves in...?
quarters	divided into halves
whole ones	half of
fractions	divided into half
divided by	

Teaching activity

Explain to the child that today's activity will help them to use mathematical words for dividing by fractions.

First, you will be showing similarities in the division vocabulary when we divide by whole numbers.

? Can you read this number sentence: $10 \div 2 = ?$

Record the words used for grouping – these are used below – comparing them with the words used to divide by fractions.

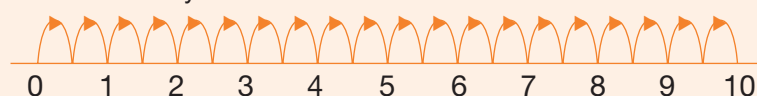
$10 \div 2 =$
 Ten divided by two.
 How many twos make ten?
 Ten divided into twos.

If the child uses some of the vocabulary of sharing, such as 'divided between' or 'shared between', you need to explain that today you are thinking of 'jumps along a number line kind of dividing' or 'grouping'.

If the child needs more support on this, you might want to draw a number line and model 'how many twos make ten'.



'Five twos make ten. How many halves make ten?'



'Twenty hops of a half make ten, because there are two halves in each whole one.'

Show the recordings from last time when you divided by fractions, for example dividing ten by a half.

We used these words when we divided ten by a half.

$$10 \div \frac{1}{2}$$

Ten divided by a half.

How many halves make ten?

Ten divided into halves.

Put the two sets of dividing words which you have recorded side by side to show the similarities in the words we use.

‘When we divide by numbers smaller than one (fractions) we can use the same words as when we divide by a number that is one or more – a whole number, such as two or four or ten. This dividing by a fraction can be worked out on a number line, just like dividing by a whole number.’

? Can you read me this number sentence: $8 \div \frac{1}{2}$?

? Now can you say the sentence in a different way?

? What is the answer? (*Sixteen halves make eight.*)

If the child needs more help, return to some of the examples used last time and use different numbers, for example, twelve cakes divided by a half, or ask the child to make up their own examples. You can record for them.

Each time, make sure that the child is using appropriate words.

You could highlight important words with a bright highlighter pen.

‘Now we are going to make some pictures in our head for two different number sentences.’

? What is half of ten?



? How many halves are there in ten? How could you draw that?

Help the child to draw either a number line showing twenty halves, or ten buns divided into halves.

Encourage the child to think very carefully about what these two different sentences mean.

Put the two number sentences together with their pictures, to compare them.

If the child needs help, they could use ten of any suitable resource and show that half of ten is five, then use ten apples and cut them in half to show there are twenty halves in ten.

‘We need to be very careful about how we use words, and we need to find a good mental picture to fit with the words.’

Some of the highlighted words could go up on display in the classroom. The recordings will be needed for *Spotlight 2*.

Spotlight 2

Interprets division as sharing but not as grouping (repeated subtraction) so is unable to interpret a calculation such as $12 \div \frac{1}{2}$

Opportunity for: making generalised statements

.....

Larger or smaller?

Time 15–20 minutes

Resources

- Some earlier recordings
- Calculator

Key vocabulary

halves	divided by
quarters	how many halves in...?
whole ones	divided into halves
fractions	

Teaching activity

‘We are going to look at what happens to numbers when you divide by a whole number, and how this is different from dividing by a fraction.’

? If we divide ten by two, is that dividing ten by a whole number or by a fraction?

? Can you make up another division by a whole number? (For example, twelve divided by three.)

Support this division with a calculator if necessary, clarifying which are whole numbers.

? When you divide by a whole number, is the answer larger or smaller than the number you started with?

Record:

$$10 \div 2 = 5$$

$$12 \div 3 = 4$$

$$20 \div 4 = 5$$

When you divide a whole number by a whole number (two or more), the answer is smaller than the number you started with.

Let the child explore on a calculator what happens when you divide by one.

Look back at previous recordings, for example from Spotlight 1.

? What do you think happens to a whole number when you divide it by a fraction (less than one), such as dividing by a half or a quarter?

Support this by looking back at previous recordings, helping the child to identify calculations that are divisions by fractions (halves or quarters, etc.) and comment on what they notice.

$$10 \div \frac{1}{2} = 20 \text{ (How many halves in ten?)}$$

$$4 \div \frac{1}{4} = 16 \text{ (How many quarters in four?)}$$

$$12 \div \frac{1}{2} = 24$$

? When you divide by a number less than one, is the answer smaller or larger than the starting number? (Larger)

If the child is uncertain, briefly review the previous activities, particularly the practical tasks.

Let the child illustrate and explain using practical equipment, showing that when you divide by a fraction the answer is larger than the starting number.

'Let's write two sentences to show what happens when you divide by a whole number, compared with dividing by a fraction. We can take what we write back to class to show the others.'

When you divide by a number less than one (a fraction) the answer is larger than the starting number.

But when you divide by a whole number (two or more) the answer is less than the starting number.

? If you worked out twelve divided by a quarter, will the answer be more or less than twelve? (More)



? Using only the digits 1, 2, 3 and 4 (you can use these digits to make fractions) write two division calculations: one with an answer which is more than the number you chose to start with; and one with an answer which is less than the number you chose to start with.

Display the two general statements from the previous page in class. 'Do you think these two statements are true? Are they always true?' (Give the children calculator experience of dividing by one and by zero.)

? What happens when you multiply a number by a fraction?

Spotlight 3

Interprets division as sharing but not as grouping (repeated subtraction) so is unable to interpret a calculation such as $12 \div \frac{1}{2}$

Opportunity for: developing mental images

3 Y6 $\times$ $\div$

Draw it

Time 15–20 minutes

Resources

- Cuisenaire or other rods
- Paper fraction pieces or *Fractions* (Resource sheet 42)
- Apples and other items to divide, or dough
- Chocolate bars or pictures

Key vocabulary

- | | |
|------------|------------------------|
| halves | divided by |
| quarters | how many halves in...? |
| whole ones | divided into halves |
| fractions | |

Teaching activity

Show the child the equipment.

‘We are going to work on dividing numbers by a half and by a quarter and I want you to draw what we do, or make a picture with anything on the table – or you can get anything else you need if you prefer.’

**? At her party, Jane has allowed a quarter of a pizza for each person.
How many quarters can she get from three pizzas?**

? Can you draw a picture of that?

? Can you tell me what to write to make a correct division number sentence?

$$3 \div \frac{1}{4} = 12$$

She can make 12 quarters.

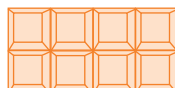
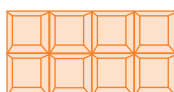
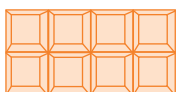
? Can you show three divided by a quarter using a different kind of picture, or with some of the equipment? (For example, choosing a rod to represent one whole one and using three of those whole ones and twelve of the related quarter rods.)

If the child seems stuck, let them make models with the dough, or with paper and encourage them to try to ‘see’ calculations in several different ways.

? Can you choose something from the equipment, or draw a picture, to show six divided by a quarter or how many quarters are there in six whole ones?

Tell me about your pictures.

Show the child some chocolate bars or pictures.



? Can you make up a division calculation with these chocolate bars, dividing by a fraction?

(There are several ways to do this: dividing by halves, quarters, eighths.)

If that is a bit too challenging, suggest which fraction they could divide by, for example halves, then give the child a while to think about it.

If they need further help, you could start a sentence for them. 'Four chocolate bars divided into halves...' (This could continue with something like 'how many children can have half a bar?')

? If the answer to one of my dividing by a fraction calculations was sixteen, what could the question have been? *(There are several possible questions, for example how many quarter apple pies can I make from four whole ones?)*

? Can you draw a picture of what you had in your head when you worked that out?

Depending on the child's drawings and the range of images they can use confidently, or the range of equipment they can work with, you might want to repeat this Spotlight before you move on.

? What is your favourite way to work out dividing by a fraction?

Spotlight 4

Interprets division as sharing but not as grouping (repeated subtraction) so is unable to interpret a calculation such as $12 \div \frac{1}{2}$

Opportunity for: reasoning about numbers

3 Y6 $\times$ $\div$

Calculator challenge

Time 15–20 minutes

Resources

- Number cards and symbol cards (Resource sheets 1, 2, 8 and 27)
- Previous recordings
- Calculator

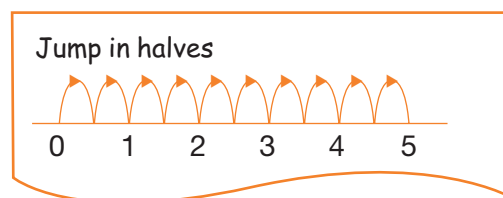
Key vocabulary

halves	how many halves in...?
whole ones	divided into halves
fractions	multiplied by
divided by	

Teaching activity

‘We are going to do some more dividing by fractions, so that you will feel confident that you can work these out.’

Choose an earlier recording or draw a number line like this.



? Tell me a division that this number line picture shows. (For example, ‘How many halves make five?’)

Support the child by counting the hops. So that there are ten hops of one half, the number line is to five and each hop is a half.

Lay out the relevant cards.

? Can you use some of these cards to show a division that matches the number line picture?

10	$\frac{1}{2}$	5	$\div$	$\times$	=
----	---------------	---	--------	----------	---

? Can you show this another way? ($5 \div \frac{1}{2} = 10$, $5 \div 0.5 = 10$)

Help the child to key in the first two numbers on a calculator but not to press the

=

 key yet.

? Does the answer get bigger or smaller when you divide it by a number less than one?

**? Do you know which number is going to come on the screen when you press the

=

 key?**

You might need to demonstrate and talk through what the calculator has done, linking it to the number line picture. If the child still finds this difficult, move on to a calculator challenge, giving the child plenty of time and examples (and plenty of opportunity to talk in order to make sense of what is happening).

'Now I'm going to give you some challenges with the calculator and you have to guess what the answer will be before you press the $=$ key.'

'Key in six divided by nought point five.'

? Can you work out which number will show when you press the equals key?

Repeat with: $8 \div 0.5$, $10 \div 0.5$, $12 \div 0.5$.

? Can you continue the pattern?

Give plenty of further examples, continuing this in your next session if the child needs more help to identify the patterns and to work out the answers themselves.

Examples could be:

- multiples of four divided by a quarter, for example $4 \div 0.25$;
- more even numbers divided by 0.5.

? What do you think will happen if you divide odd numbers by a half? (*The answer is double the starting number, just like even numbers.*)

You might want to record some of their calculations, but make sure you give them plenty of time when you are not looking at the display so that they can try things out for themselves.



? Can you explain why this division and this multiplication have the same answer?

? Can you make another two calculations that link together in a similar way?

$$4 \div 0.25 = 16$$

$$4 \times 4 = 16$$

Spotlight 5: a learning check

Interprets division as sharing but not as grouping (repeated subtraction) so is unable to interpret a calculation such as $12 \div \frac{1}{2}$

Opportunity for: discussing and explaining

Picture match game

Time 10–15 minutes

Resources

- Picture match game board (Resource sheet 43)
- Picture match game cards (Resource sheet 44)
- At least two players
- Calculators
- Rewards such as counters, stickers, etc.

Check: does the child use key vocabulary?

- | | |
|------------|------------------------|
| halves | how many halves in...? |
| quarters | divided into halves |
| whole ones | hops of a half |
| fractions | divided into half |
| divided by | half of |

Teaching activity

Put out the game board (Resource sheet 43) for **Picture match game** and give the children a chance to look at it carefully and to count the jumps, and so on. Encourage them to talk to each other about what they see.

Put out, face up, an even number of the picture match cards from Resource sheet 44. (If you wish, you can just use the cards in the left-hand column of the sheet. These all go with pictures 1 and 3. The cards in the right-hand column all go with the pictures, except for the last one, which doesn't match any of the pictures.)

How to play

1. The players take turns to pick a card, read it to the other player and decide which picture that number sentence can go with. (Some sentences will go with more than one picture. Some sentences might not go with any of the pictures.)
2. If everyone agrees that they are right, the player will win a reward. If they aren't right, the card then passes to the other player, and, if they are right, they get another turn.
3. Take turns until all the cards are used up and the player with the most rewards wins.

? How did you decide that card went with that picture?

? Will that number sentence go with any of the other pictures as well?

Variations

- Pairs can play cooperatively or play against another pair.
- Don't provide any cards but ask the players to make cards for all four pictures and then give the cards to another pair.
- Let the children make four pictures and related cards of their own, which they should give to another pair to sort out.

Learning outcomes

By the end of this set of activities, children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- interpret a calculation in which there is dividing by a fraction;
- explain their drawings and mental images when dividing by fractions;
- solve real-life problems involving division by fractions;
- begin to generalise about number patterns when dividing by a number less than one.

Is not confident in making reasonable estimates for multiplication or division calculations

Opportunity for: making decisions



Resources

- Tape measure
- Number lines
- Calculator

Key vocabulary

estimate	almost
guess	nearly
rough answer	

Teaching activity

Time 15–20 minutes



Explain to the child that today's activity will help them to make an estimate of a calculation.

'Tell me what you think making an estimate means.'

? Give me an example of making an estimate.

If the child is unsure about this, give an example.



'When we are measuring, we often make an estimate or a guess. So I could estimate how far it is across our table. My guess is that it is about (eighty centimetres/or other). My guess probably isn't the exact measurement – it is only an estimate, or a guess, or sometimes we can call it a rough answer.'

? How could we find out if my guess is right?

Help the child to measure across the table.



'My estimate was close/not very close. But it was just a guess: an estimate.'

'Estimating can help us when we are doing calculations too. So if we know that a hundred divided by two is fifty, if we have to work out a hundred and seven divided by two, we can make an estimate of the answer. The answer must be somewhere close to fifty.'

Let the child work it out on a calculator.

'So fifty-three point five is quite close to fifty. Fifty was our estimate. Fifty-three and a half was the exact answer.'

You might find it helpful to use a number line to 100, marking 100, 50 and 25.



? Which do you think would be the best estimate of fifty-six divided by two: twenty-six or six?

Help the child to think through dividing fifty by two to get to twenty-five, so twenty-six is a much closer estimate than six.

You might want to record some calculations to take back to class or to refer to later on in this work. For example:

An estimate of 17×2 is about 25.

Or 17×2 is about 30.

? Which is closer to the actual answer, 25 or 30? How do you know?

Help the child to explore their thinking. Again, a number line might help. Show how $17 \times 2 = 34$, so 30 is a much closer estimate than 25 on the number line.

If the child seems lost and is not grasping what an estimate is, you might find that it helps to go completely over the top and write some calculations that they will see are nonsense!

12×3 is about 14 562

$10 \div 2$ is about 376

10×2 is about 968

? How do you know these are not close estimates?

‘So when we make an estimate, we don’t need to get the answer right, but we do need to make a close guess to try to find an estimate that is close to the actual answer.’

? If we wanted to make an estimate of thirteen multiplied by nine, which would be the closer estimate: $10 \times 10 = 100$ or $20 \times 20 = 400$?



Again, a number line would be useful. Thirteen can be seen to be quite near ten, but further from twenty. Nine is also near ten, so ten multiplied by ten is likely to give us an estimate which is nearer the actual answer than twenty multiplied by twenty.

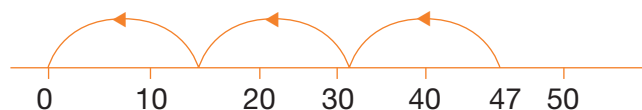
Support the child to calculate thirteen multiplied by nine in any way they want. For example, $10 \times 9 = 90$ added to $3 \times 9 = 27$ that is 117 altogether.

? So was our estimate close to the answer?

? If we wanted to make an estimate of forty-seven divided by three, do you think the answer would be nearer to ten or nearer to a hundred? Why?

You might want to find 47 on a number line and help the child to think about roughly what size each of the three hops back might be.

? If we make a guess at the size of each of the three hops back from 47, about how big is each hop? Make a guess.



? Is each hop nearer to a hop of ten or a hop of a hundred?

? Is each hop a little bit bigger than ten or a little bit smaller than ten? (Bigger)

47 ÷ 3 is somewhere around ten, but a bit more than ten.
We worked it out and it is 15 and a bit, so we made quite a good estimate.

? What shall we record from today’s lesson that is really important to remember for next time?

(Note: Children who think that mathematics is only about getting the one right answer can be confused by estimating – presumably because we are accepting their guesses and saying those are ‘right’! And we tell them not to work it out exactly at first! So you might find you have to repeat this lesson and, crucially, emphasise making an estimate of a calculation *every time* we do a calculation, and then using the estimates after calculating to check that our answer is a reasonable one.)

Spotlight 1

Is not confident in making reasonable estimates for multiplication or division calculations

Opportunity for: solving real-life problems and reasoning about numbers

.....

Estimating with multiplication tables **Time** 15 minutes

Resources

- Cubes
- Ten paper ‘tables’
- Number lines
- *Multiplication grid 1* (Resource sheet 35)
- Calculator

Key vocabulary

estimate	about right
guess	close but too big
rough answer	close but too small
almost	round up/down
nearly	

Teaching activity

‘We are going to do some estimating today. Sometimes we will be working out answers exactly, but first we are going to make a good guess, an estimate, and we are going to do just a little rounding of numbers.’

(Note: Take care when talking about rounding. During trialling, when asked to round numbers, some children who were asked to find an approximate calculation for 49×2 stuck so closely to rules of rounding that they rounded the calculation to 50×0 ! Not a good estimate!)

‘The teacher has thirty-two children in her class and they sit eight to a table. Shut your eyes and think about those numbers and try to see the children in the class and the tables.’

Show the pieces of paper representing tables. ‘Eight sit at this table, eight more at this table...’

? About how many tables do you think they will need?

Give the child time to think. Let them choose something to help – but not a calculator. They could put eight cubes around one of the pieces of paper and look at how many cubes are left for the other tables.

If the child seems stuck, you could try asking them to think of an easy number near thirty-two, probably thirty.

'We can round thirty-two down to thirty.' Use a number line to show how close 32 is to 30.

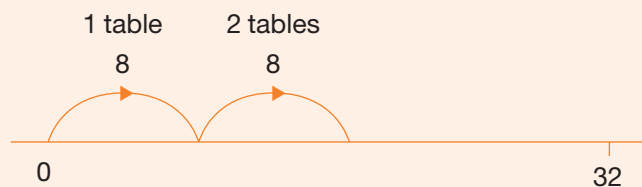
? About how many eights are there in thirty? So do you think an estimate of five or ten tables is the closest?

Or talk through the fact that ten tables with eight to a table would seat eighty children!

If the child knows that $4 \times 8 = 32$ and can see that four tables are needed, move on.

If the child needs more help, let them do a recording in any way they choose to show how they worked out the calculation.

If they are stuck, suggest they use a number line and see how many hops of eight they need, starting either from zero and counting up, or starting from 32 and counting back.



'You need four hops of eight to get to 32. So five was the closest of the two estimates.'

'We found $4 \times 8 = 32$ on the multiplication grid.'

You can record for the child. Support the child to work mentally at first, using a number line and/or a multiplication grid, and later let the child use a calculator to try each calculation.

'We know that thirty-two divided by eight is four: $32 \div 8 = 4$.'

? What if the tables only seated six children? Would thirty-two divided by six give a bigger or a smaller answer than thirty-two divided by eight?

(Record what the child thinks. The answer is larger because steps of six are smaller than steps of eight so you need more of them. The child might explain that you would need more tables if they only seated six children.)

? What if the teacher only had twenty-four children and it was eight to a table?

(Record for the child. The answer is 'smaller' – the teacher would need fewer tables with fewer children.)

If the child is struggling with this, you will need to repeat this lesson, because being able to reason about numbers in order to make estimates is so important. You can use numbers to suit the child.

Also try some more over-the-top calculations.

Do you think $100 \div 2 = 3$? How do you know?

Is this right: $94 \div 6 = 5$? How do you know?

? Is eighteen multiplied by nine larger or smaller than eighteen multiplied by eight?

In a real-life context that is:

- ? Are there more marbles in nine bags or in eight bags?
- ? What shall we record today that is really important to remember?

Spotlight 2

Is not confident in making reasonable estimates for multiplication or division calculations

Opportunity for: making decisions

Rounding to estimate

Time 15 minutes

Resources

- Number lines
- 100-square
- Bead string
- Calculator

Key vocabulary

- | | |
|--------------|---------------------|
| estimate | about right |
| guess | close but too big |
| rough answer | close but too small |
| almost | round up/down |
| nearly | |

Teaching activity



'We are going to do some more estimating today, using rounding.'

- ? Can you remember what rounding is?
- ? How could you round forty-nine multiplied by two to help find an estimate of the answer?

Clarify that forty-nine can be rounded to fifty – but we wouldn't round the two!

- ? What is fifty multiplied by two?
- ? Do you think that sixty is a good rough answer for forty-nine multiplied by two? Why not?

You could show two hops of forty-nine on a number line. Or show 49 and 50 on a hundred square, or on a bead string.

- ? If I have fifty-nine pence, have I got nearly sixty pence or nearly seventy pence? How do you know?
- ? How could we estimate fifty-nine divided by three? $59 \div 3 = (60 \div 3 = 20)$
- ? Can you make up a number story for fifty-nine pence divided by three? (Fifty-nine pence divided into three equal groups so that three children can have one-third of the money each.)

You could use a bead string to show the rounding to sixty, then find how big the three hops back from 60 to 0 are.

But there is only fifty-nine pence! So there isn't enough for each child to have twenty pence. Twenty pence was only an estimate – not the actual answer. The estimate of twenty pence helps us to check the actual answer.

? What do you think the actual answer is? It must be very close to twenty pence.

Estimating helps us to check our answers.

? When we work out forty-seven multiplied by two, which number would we round up or down to help us find an estimate? (*Forty-seven probably best rounded up to fifty.*)

? Would you need to round the two? Why?

? How could we find an estimate for five hundred and ninety-nine multiplied by two?

? Would you round the two?

? What about seventy-eight multiplied by thirteen? What might be an estimate for that? How did you decide?

On a number line you could show how 78 is close to 80 and 13 is close to 10.

'This time we could round both numbers to make the estimating easier to do. We rounded 78 up to 80 and we rounded 13 down to 10.'

Let the child work out 80×10 , record the answer, then work out on a calculator the actual answer to 78×13 . Mark both numbers on a number line.

Give another example of rounding one number up and one down, for example: 73×9 .

Round seventy-three down to seventy. Round nine up to ten.

$$70 \times 10 = 700$$

? What shall we record that is important to remember for next time?

Support the child as they reflect on what they have learned.

Keep this recording for the next session.

Spotlight 3

Is not confident in making reasonable estimates for multiplication or division calculations

Opportunity for: making decisions and explaining

4 Y6 $\times$ $\div$

In-between

Time 10–20 minutes

Resources

- Selection of number lines
- 100-square
- Calculator

Key vocabulary

estimate	about right
guess	close but too big
rough answer	close but too small
almost	round up/down
nearly	which is closer?

Teaching activity

‘Let’s look back at what we wrote last time. We wrote down something very important to help us remember ways of finding estimates.’

Go over what you wrote.

‘Today we are going to look at another strategy that can really help us when we want to estimate an answer. We can do two estimates so that we know that our actual answer must be somewhere between those two estimates.’

? How could you estimate the answer to two hundred and fifty-seven multiplied by three?

Support the child to see that you could round 257 up to 260 or down to 250. (We won’t round the three!)

If the child is struggling with this rounding, use a number line from about 250 to 300 to show the rounding to numbers with zeros, and extend the line to about 800 to show where the rounded answers and the actual answer fit on the line.

257×3

Making one estimate above and one below the actual answer (771) can help us to be sure our answer is correct.

So if you work out 257×3 and get 1078 you know you are wrong!

Ask the child for another three-digit number to round both up and down so that the answer is between the two, for example 386×2 . (This could be rounded up to $390 \times 2 = 780$ and down to $380 \times 2 = 760$. The actual answer is 772.)

? What did you learn today?



(You can make estimates every time you calculate anything because they help you to get more answers correct. **Estimate, calculate, check.**)



? How could you work out an estimate for 555×5 ?

Spotlight 4

Is not confident in making reasonable estimates for multiplication or division calculations

Opportunity for: generalising about numbers

More or less?

Time 15 minutes

Resources

- Squared paper
- Cubes
- Selection of number lines
- 100-square
- Calculator

Key vocabulary

estimate	about right
guess	close but too big
rough answer	close but too small
almost	round up/down
nearly	

Teaching activity

'We are going to do some estimating answers today, and we will be trying to work out whether the estimates we make will be more or less than the actual answers. We are going to start with multiplying.'

? Which is more, three multiplied by twelve or four multiplied by twelve? How do you know?

? Can you choose something on the table to show me how you know?

Encourage the child to make or draw an array on squared paper or with cubes. You could record this.



3 multiplied by 12

Three multiplied by twelve
means twelve columns of three.

Four multiplied by twelve would need twelve columns of four, so 4×12 is more than 3×12 .

If the child has forgotten about arrays, make some on squared paper with them and stick them up somewhere so that the child can be reminded about them over the next few days and weeks.

Try some more examples.

? Which is larger, twenty-seven multiplied by five or twenty-seven multiplied by four? Can you tell me how you know that?

Let the child work out the actual answers using a calculator, because the point of this lesson is to be able to have lots of experience of comparing answers. Using calculators can take the worry out of finding answers and it allows children to concentrate on understanding the mathematical relationships.

Record for the child:

27×5 is more than 27×4 .

16×6 is more than 16×5 .

46×9 is more than 46×8 .

You could use colour to emphasise the five and the four in the first calculation, then another colour to emphasise the six and the five, and so on.

? Can you see a pattern in the coloured numbers?

Look at each example carefully and try to move to making a generalisation.

27×5 is more than 27×4 .

16×6 is more than 16×5 .

46×9 is more than 46×8 .

When we multiplied these, we found a pattern: the larger the numbers you multiply by, the larger the answer.

27×5 is more than 27×4 .

To support the child, you could use numbers that are further from each other, for example 8×5 is less than 8×10 .

Let the child draw or make arrays and calculate the answers, looking for the pattern that the larger the numbers are, the larger the answer will be.

? Can you tell me how you know that the larger the numbers you multiply by, the larger the answer will be?

‘Let’s try to see if we can find a pattern when we divide.’

? Predict what might happen when we divide eight by *two* and when we divide eight by *four*.

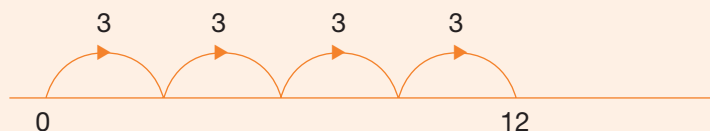
? Which will give the smaller answer: eight divided by two or eight divided by four?

? Thirty-two divided by two or thirty-two divided by three?

? Twelve divided by three or twelve divided by four?

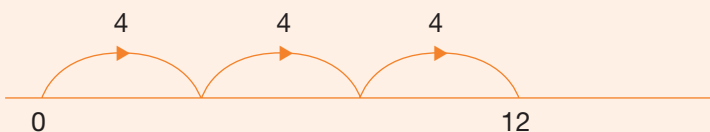
? Twenty-four divided by six or twenty-four divided by eight?

Remind the child that twelve divided by three means how many threes make twelve, and we can show this on a number line as 12 with equal steps of three. Each step will be a step of three and there are four of them.



Twelve divided by four means how many steps of four are there in twelve?

There are three steps of four in twelve.



$$12 \div 3 = 4$$

$$12 \div 4 = 3$$

Support the child to moving towards making a generalisation.

Which will give the smaller answer:

$$32 \div 2 \text{ or } 32 \div 3?$$

$$12 \div 3 \text{ or } 12 \div 4?$$

$$24 \div 6 \text{ or } 24 \div 8?$$

The smaller the number you divide by (the step size), the larger your answer.

The smaller the size of the steps along the number line, the more of them you need (a larger number of steps).

The larger the number you divide by (the step size), the smaller your answer.

‘Let’s check our prediction.’

Let the child explore, using a calculator to check the generalisations about multiplication and division. You record for them.



You could take the generalisations back to the class.

(Note: This kind of activity using a calculator can help children to develop a strong understanding of place value.)

This would be a good time to play the game **In one move**, which can be found on page 16.

Spotlight 5

Is not confident in making reasonable estimates for multiplication or division calculations

Opportunity for: discussing mental methods

4 Y6 $\times$ $\div$

Be careful!

Time 15–20 minutes

Resources

- Cubes
- Squared paper
- Selection of number lines
- Calculator

Key vocabulary

estimate	about right
guess	close but too big
rough answer	close but too small
almost	round up/down
nearly	

Teaching activity

‘This lesson is about making estimates of calculations first, then using the estimate to find an exact answer.’



? Can you think of a way of estimating nineteen multiplied by four?

? Make your estimate and tell me how you’ve done it.

A calculation the child might suggest is 20×4 .

Encourage children not to round the four to five, because that would make the estimate a long way from the actual answer. They can multiply by four by doubling the twenty to forty and double again to get to eighty, or use another mental method they like.

? Can you use your rough answer, your estimate, to find the exact answer in your head?

Listen carefully to what the child says. They may be making the error shown below.

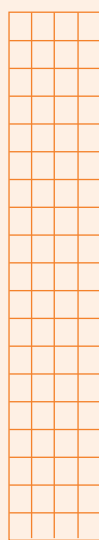
Note: In trialling, children sometimes used mental methods more appropriate for addition and subtraction calculations. So this Spotlight focuses on this.

If the child says anything along the lines of: ‘We rounded up from nineteen to twenty so to find the real answer I can just take away one, so the actual answer is $80 - 1 = 79$ ’ then the child is using addition and subtraction mental strategies! You might want to record these because they do need talking through.

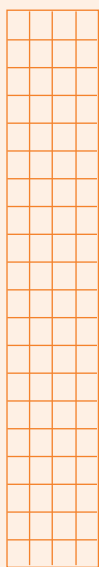
19×4 is close to $20 \times 4 = 80$
So to find the exact answer I can do $80 - 1 = 79$
because we added one to get from 19 to 20.
But that is wrong.
 $19 \times 4 = 76$ not 79.
Where did we go wrong?

Let the child work out the actual answer using a calculator and record the results for them.

Support the child to use squared paper or cubes to see the difference between an array of four columns of nineteen and four columns of twenty. There needs to be one extra row of four, four more cubes on the array.



$$19 \times 4$$



$$20 \times 4$$

'There is one extra row of four cubes here.'

'So subtracting one was wrong. I think you probably meant to subtract one row of four. You were using a mental method used with addition and subtraction, not with multiplying and dividing.'

'When we estimate in multiplying and dividing, each time we round up or down, we add on or take off columns or rows.'

Give another example, such as an estimate for 26×3 .

? When we are estimating calculations with multiplication and division and then trying to find the actual answer, is it okay to use the same mental methods that we use for addition and subtraction? (No) How do you know that?

? What shall we write down for today that is important to remember?



Why do you think it is important to be able to make estimates?

Encourage children to make estimates of every calculation they do!

You could put the following on the wall:

Estimate
Calculate
Check
Every time!

Spotlight 6: a learning check

Is not confident in making reasonable estimates for multiplication or division calculations

Opportunity for: explaining and discussing

Which side?

Time 10–25 minutes

Resources

- At least one other child, four is ideal
- Counters
- Bag with number cards 1–9 (Resource sheet 1)
- Another bag with two cards: ‘more than’ and ‘less than’
- Spinners (Resource sheet 44)
- Blank spinners (Resource sheet 13)
- Paper clip and pencil
- Selection of number lines
- Calculator

Check: does the child use key vocabulary?

estimate	about right
guess	close but too big
rough answer	close but too small
almost	round up/down
nearly	

Teaching activity

‘This game, **Which side?**, will help you to get better at estimating, and we are going to use a number line to put our estimates on and compare them with the actual answer.’

You might want to put on display the recordings from previous Spotlights of things that needed to be remembered.

Players can work cooperatively in teams or in pairs, or they can compete on their own against another player.

The spinner is worked by trapping the paper clip in the middle of the spinner with the point of a pencil. With the other hand, flick the paper clip around. If the paper clip lands on a line, that player chooses which number they want.

Spinners work best if they are copied onto card.

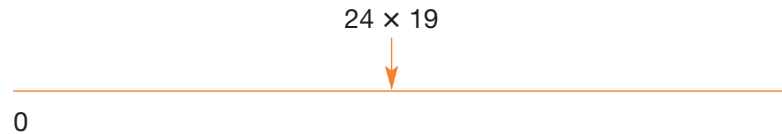
Before you start, decide which spinner you are going to use. If you cut out your spinner, be sure to cut in along the boxes in order to leave enough space for the spinner to be held still while it is being used.

Blank spinners (Resource sheet 13) can be used for any numbers you choose.

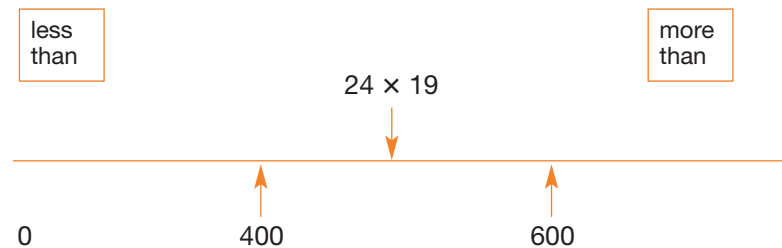
How to play

1. A player takes two digit cards out of the bag and makes a two-digit number, for example 24. This number is then recorded and is used for both teams. The digit cards are put back in the bag.
2. Another player, on any team, flicks the paper clip to choose the operation and number, for example $\times 19$.

- Each team then records the calculation on paper, in this case 24×19 , and someone draws a number line and marks the calculation.



- Each team then takes one card out of the other bag and shows it to the group.
- The team with the 'more than' card must make an estimate of the calculation 24×19 that will give an answer which is more than the actual answer. The team with the 'less than' card must make an estimate which is less than the actual answer.
- Each team then works quietly together to work out their estimate.
- When everyone is ready, the teams take turns to tell everyone about their estimate.
 'We had to make an estimate that would be more than 24×19 , so we chose 30×20 which is 600.'
 'We had to make a rough answer less than the actual answer, so we estimated 20×20 which is 400.'
 The teams can challenge any estimate which they think isn't sensible, for example 40×30 would not be a sensible estimate for this calculation.
- Both teams can put their estimates where they think they go on the number line.



- Then everyone has to work out the actual answer. (Calculators will speed this up and help to take the stress out of the game.)
- Each team should keep a record of their score. For each round they get:
 ten points for a sensible estimate;
 ten points for calculating their estimate correctly;
 ten points for estimating 'more than' or 'less than' correctly;
 a ten point bonus for calculating the actual answer without a calculator.
- Then a new two-digit number is made and another number selected on the spinner.

- ? How did you work out that your estimate would be more than the actual answer?**
- ? Whose estimate was the closest to the actual answer?
How do you know?**
- ? What if that team had used (a number) instead of (a number)? Would their answer have been closer to the actual answer or further away?**
- ? If we draw that number line again, could we position our estimates a bit more accurately?
(Some estimates will be much closer to the actual answers, depending on the numbers chosen.)**
- ? Which team got the closest to the actual answer?**
- ? What shall I record for you to remember for next time?**

Variations

- Play as **Who is the closest?** Players compete to find an estimate as close as they can to the actual answer. (So you don't need to play with the 'more than' and 'less than' cards.)

The winning team scores ten points, or if they are both the same distance on the number line from the actual answer, they score ten points each. (You will need to help the players to decide who is the closest when they have two different answers. Encourage the players to round numbers to fives as well as numbers with a zero at the end, so fifty-four might be 'rounded' to fifty-five to give a closer estimate.)

(Note: Watch out for children rounding all the numbers! On Spinner 2 hopefully no one will round $\times 4$ to $\times 0$!)

- ↑ ● Play **Who is the closest?**, as above, but also play with the 'more than' and 'less than' cards. This can be very tricky, and children will realise that if they have the 'more than' card, but the numbers are, for example 21×51 , they will be a long way from the actual answer if they round up to 30×60 .

? Can you think of a way to estimate 21×51 that might be closer? (For example 25×50)

? What if you round one number up and the other number down? Does that give you a closer estimate? (It can do.)

? Tell me about an example when this happens.

- ↑ ● Make three-digit numbers at the start.
- Use the blank spinner to make up your own numbers to multiply and divide. (The game can be played by estimating with addition and subtraction but clarify with the players that they must be careful in the way they go from their estimate to the actual number. What we do for addition and subtraction needs rethinking to find a method that might work for multiplication and division!)

Learning outcomes

By the end of this set of activities children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- make a reasonable estimate of a multiplication or division calculation including calculations with fractions;
- compare two multiplications or divisions and know which one will give the larger answer;
- understand that an estimate is a rough answer;
- make generalisations about the effects on numbers of multiplying and dividing by both numbers more than one and numbers less than one;
- understand that it helps accuracy to make an estimate for all calculations that cannot be calculated mentally immediately;
- understand that in real problem-solving we have to make sensible decisions;
- position estimates and actual answers on a number line.



In one move

Time 15–20 minutes

Is not confident in making reasonable estimates for multiplication or division calculations

Resources

- Children in pairs (or use with the whole class)
- Calculator for each child

Check: does the child use key vocabulary?

number pattern more than
less than

How to play

- Set a task, for example:

‘Key in 200. In one move (one operation, either $\times$ or $\div$, a number and the $=$ key) make your display read 20.’

Children can press cancel at any time if they want to – but not so that they can just key in 20!

- Ask children what they did. You can record for them.

If you multiply 200 by 2 the answer is 400 so the answer is bigger than the 200 we started with.

But, if you multiply the 200 by a number less than 1, for example 0.5, you end up with an answer (100) that is less than the 200 you started with.

So using the $\times$ key doesn't always make the answer larger.

- Repeat with the $\div$ key.
- Once children are used to the game, you can introduce a scoring method, maybe 0.1 for each time a player does the task ‘in one move’.
- Encourage children to look for patterns.

$10 \times 0.1 =$	$100 \div 0.2 =$
$100 \times 0.1 =$	$10 \div 0.2 =$
$1000 \times 0.1 =$	$1 \div 0.2 =$

Variations

- Encourage children to make up challenges for each other, for example, start with 10 on the screen and just use the division key. Can you in one move make the screen read 100?
- Challenge children to make a long pattern using similar numbers, for example: If you start with 3000×0.3 , can you make patterns using 300, 30 and 3? What if you start with 300 again but this time $\times$ by 0.1?

Learning outcomes

By the end of a set of related activities children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- understand the effects of multiplying and dividing by numbers less than one.